



Fig. 4

In the derivation of the values of  $\sin A$  and  $\cos A$ , we will first need to know the direction angles  $a, b,$  and  $c$ . We have already determined angle  $a$  as  $\cos^{-1} dp/dx$ .

The relationship of the increase in path length to the distance between average recording points will hold in the same manner for the "T", and this relationship is  $dp/dy$ .

It is apparent from Fig. 4 that angle  $b$  is  $\cos^{-1} dp/dy$ , where  $b$  is the angle which the line  $w$  makes with the Y axis.

Since we cannot determine angle  $c$  from observed data, we must derive its value from the other two, keeping in mind that angle  $c$  is the angle which the line  $w$  makes with the vertical or Z axis.

From analytic geometry the relationship of the direction angles is:

$$\cos^2 a + \cos^2 b + \cos^2 c = 1$$

then:  $\cos c = \sqrt{1 - \cos^2 a - \cos^2 b} \dots\dots\dots(12)$

After determining the cosines of the direction angles, it follows from Fig. 3 that:

$$\left. \begin{aligned} Yd &= w \cos b \\ Zd &= w \cos c \\ Xd &= X - w \cos a \end{aligned} \right\} \dots\dots\dots(13)$$

Also from Fig. 3 it is apparent that  $\sin A = \frac{MR}{LR}$

but  $MR = Yd$  and  $LR = \sqrt{(MR)^2 + (LM)^2}$  or  $\sqrt{Yd^2 + Zd^2}$

then  $\sin A$  becomes  $\frac{Yd}{\sqrt{Yd^2 + Zd^2}} \dots\dots\dots(14)$