Seismic Modeling, Migration and Velocity Inversion

Inverse Scattering

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Outline

1. **Fundamental Principles**
2. **Green's Functions, Operators, Data and Sources**
   - Functions
   - Operators
   - Data and Sources
3. **Full Waveform Inversion**
   - Operator Form
4. **Inverse Scattering**
   - The Lippmann-Schwinger Equation
   - The Forward Scattering Series
   - The Inverse Series
   - Convergence
   - Multiple Elimination
   - Internal Multiple Elimination
   - The Final Step
5. **Final Comments**

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Fundamental Inversion Principle

- The fundamental principle underlying the inversion of seismic data is the determination of an Earth model that is fully consistent with measured data. That is, given data U measured over some part of the Earth's surface, inversion seeks that Earth model whose synthetic seismic response matches the measured data as accurately as possible.

Full Waveform Inversion

- Full waveform inversion is a Newton-Raphson style minimization method. (see e.g. Lailly (1983), Tarantola (1984), Crase (1990), Mora (1987,1999), Pratt (1999), and Pratt and Shipp (1999)) A quantitative difference between measured, for which the underlying mathematical model is not known, and synthetic data, for which a model is known, is minimized.
- If successful this approach provides a model that is completely consistent withe observed or measured data.

Inverse Scattering Inversion

- Inverse scattering is a step-by-step method in which parts of the original measured data are eliminated until the remaining data consist of primary reflections only. The final step seeks to produce an accurate image with zero knowledge of the primary Earth model.
- Each step involves identifying and summing a sub series of the full inverse scattering series. There are, of course, sub series sums that focus on other useful information, but these are beyond the scope of this presentation.

Full waveform Inversion and Inverse Scattering

- Alternative approaches to seismic data inversion.
  - FWI minimizes an objective function based on the difference between synthetic and real data
    - Some have called this an indirect method
    - Suggests that it is not worth the research effort it currently enjoys.
  - Inverse scattering is a step-by-step approach the removes all multiples before producing an image of the seismic data without knowledge of the velocity
    - Some consider this to direct method
    - As such it is superior to FWI
- At first glance the two approaches seem to have sufficiently different theoretical foundations to suggest that they have little in common,but as we will see they both work with residual type data.

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Green's Function

The Green's function for a simple two-way-frequency-domain-scalar wave equation is defined as the solution to

$$\left(\frac{\omega^2}{K} + \frac{1}{\rho}\nabla^2\right) G(\mathbf{r}, \mathbf{r}_s, \omega) = -\delta(\mathbf{r} - \mathbf{r}_s),$$

where δ is the Dirac delta function, $\mathbf{r}$ and $\mathbf{r}_s$ are the field and source location variables, K is bulk modulus, and ρ is density. In mathematical terms, G is the impulse response of the inhomogeneous differential equation.

Green's Operators

The principle of superposition says that convolution $U = G * g$ of a Green's function with a source $g(\mathbf{r}_s, \omega)$ is the solution to the inhomogeneous equation

$$\left(\frac{\omega^2}{K} + \frac{1}{\rho}\nabla^2\right) U = g(\mathbf{r}_s, \omega)$$

so we can think of G as an operator from sources to data.

Differential Equations as Operators

If we define the differential operator L as

$$\mathbf{L} = \left(\frac{\omega^2}{K} + \frac{1}{\rho} \nabla^2 \right)$$

then **L** is an operator from data back to sources and the corresponding Green's operator is the negative inverse of **L**. Thus,

$$\mathbf{LG} = -\mathbf{I}$$

Operator Relationships

In operator terms, $U = \mathbf{G}(\phi)$ for a source ϕ satisfies

$$\mathbf{L}(U) = \phi. \tag{1}$$

In this case, U when restricted to a recording surface is what we like to call seismic data and normally is the five-dimensional data set we record on or near the surface. Using modern computational resources, U can be synthesize quite efficiently and in this context measured everywhere on and within the model defining $\mathbf{L}$.

Sources as data and data as sources

In what follows, we make no mathematical distinction between U or ϕ. They are simply well behaved continuous possibly vector valued functions on the same domain. They are differential up to the highest order deemed appropriate and are elements of the same function space. However, because **G** generates data, and **L** produces sources, it is convenient to think of the first in terms of sources and the second in terms of data.

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The Optimization Problem

Full waveform inversion (FWI) is formulated as an optimization problem where an objective function of the form

$$J(U, U_0) \tag{2}$$

is minimized. Here $U = \mathbf{G}(\phi)$ represents recorded data for an unknown **L** and source ϕ and $U_0 = \mathbf{G}_0(\phi)$ represents synthetic data for a known Green's function. The determination of an accurate version of ϕ is critical for FWI to be successful. Long offsets and low frequencies are also extremely important.

The Objective Function

In the current research literature, (see Symes (2010)) the function J takes many forms. Most frequently J is the L^2 norm (Pratt (1999), Pratt and Shipp (1999)) , but it can also be an L^1 norm, the norm of a phase difference (Bednar et. al. (2007)), a logarithmic difference (Shin et. al. (2007)), an amplitude difference (Pyun et. al. (2007)), the norm of a difference of analytic functions, or the norm of envelope differences. It has been specified in space-time, frequency, and in the Laplace domain (Shin and Cha (2008)).

Earth Model Representation

Earth models in FWI are represented by a discrete set $\mathbf{p} = \{p_k\}$ of parameters consistent with what is believed to be the true subsurface and in this case, the updating scheme is expressed as

$$\mathbf{p}^{\,n} = \mathbf{p}^{\,n-1} - \mathbf{H}^{-1}_{\mathbf{p}^{n-1}} \nabla_{\mathbf{p}^{n-1}} J \tag{3}$$

Where $\nabla_{\mathbf{p}^{n-1}} J$ is the gradient of J. We note that this gradient is some form of residual or difference between the recorded and synthetic data. Traditionally, $\mathbf{H}^{-1}_{\mathbf{p}^{n-1}}$ is the inverse of the Hessian matrix. Since in most cases, $\nabla_{\mathbf{p}^{n-1}} J$ represents a migration of a residual, we prefer to view $\mathbf{H}^{-1}_{\mathbf{p}^{n-1}} \nabla_{\mathbf{p}^{n-1}}$ as a special type of imaging condition.

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In operator From

Clearly the updating scheme can also be expressed in operator form

$$\mathbf{L}_0^n = \mathbf{L}_0^{n-1} - \mathcal{H}_{n-1} \nabla_{\mathbf{p}^{n-1}} \mathbf{R}^{n-1}, \tag{4}$$

where $\mathbf{R}^{n-1}$ is the appropriate residual, but because it usually requires an excessive amount of memory this approach has never been popular. From the authors' perspective, FWI is relatively simple to explain, understand, and implement. It is not clear that a similar statements can be made relative to ISS.

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Literature

One of the very first papers on this methodology was that of Ware and Aki (1969). A good beginners reference for inverse scattering is provided by Weglein et. al. (2003). Interested readers are refereed to the latter article for more precise explanations of the topics in the following discussions.

Lippmann-Schwinger

Because it is the usual starting point it is not possible to discuss ISS without first reviewing the Lippmann-Schwinger Equation. For invertible operators or matrices $\mathbf{L}_0$, and $\mathbf{L}$ or their inverses $\mathbf{G}_0$ and $\mathbf{G}$ with the same domain and range one can write

$$\mathbf{L} - \mathbf{L}_0 = \mathbf{L}\left(\mathbf{G} - \mathbf{G}_0\right)\mathbf{L}_0$$

and

$$\mathbf{G} - \mathbf{G}_0 = \mathbf{G}_0\left(\mathbf{L} - \mathbf{L}_0\right)\mathbf{G}$$

These Lippmann-Schwinger Equations are, in fact, straightforward operator identities. There is absolutely nothing overly complex or special about them. The operators can be fairly arbitrary and have almost nothing in common. As long as they have appropriate dimensions exactly the same domain and range and are invertible, both equations hold.

The Forward Scattering Series

With

$$(\mathbf{I} - (\mathbf{L} - \mathbf{L}_0))\,\mathbf{G} = \mathbf{G}_0$$
$$(\mathbf{I} - (\mathbf{G} - \mathbf{G}_0))\,\mathbf{L} = \mathbf{L}_0$$

we have the so-called forward scattering series

$$\mathbf{G} - \mathbf{G}_0 = \mathbf{G}_0 \sum_{n=1}^{n=\infty} (\mathbf{L} - \mathbf{L}_0)^n\,\mathbf{G}_0^n$$
$$\mathbf{L} - \mathbf{L}_0 = \mathbf{L}_0 \sum_{n=1}^{n=\infty} (\mathbf{G} - \mathbf{G}_0)^n\,\mathbf{L}_0^n$$

Convergence

While neither of the Lippmann-Schwinger equations are of much use in what follows, it is worth noting that

- There is no need for uniform convergence
- They converge for some source ϕ or data U only if
  - $\| \mathbf{GL}_0(U) - U \| < 1$ and $\| \mathbf{LG}_0(\phi) - \phi \| < 1$
- In essence, $\mathbf{L}_0$ must be close to the inverse of $\mathbf{G}$ and $\mathbf{G}_0$ must be close to the inverse of $\mathbf{G}_0$
- Note that both of these formulas are based on a residual difference between either data or sources

Inverse Series

The inverse series is formed by assuming that either $\mathbf{G} - \mathbf{G}_0$ or $\mathbf{L} - \mathbf{L}_0$ can be expressed as in terms of powers of the other. Thus taking one of

$$\mathbf{V} = \mathbf{L} - \mathbf{L}_0 = \sum_{n=1}^{n=\infty} \mathbf{V}_n$$

$$\acute{\mathbf{V}} = \mathbf{G} - \mathbf{G}_0 = \sum_{n=1}^{n=\infty} \acute{\mathbf{V}}_n$$

and then solving for $\mathbf{V}_n$ or $\acute{\mathbf{V}}_n$ for each n provides a series in terms of powers of either the data or the source differences.

Solving for $\mathbf{V}_k$

For example, solving for $\mathbf{V}_k$ begins by collecting terms of like order in the series

$$\mathbf{G}_0 \sum_{n=1}^{n=\infty} (\mathbf{L} - \mathbf{L}_0)^n \mathbf{G}_0^n = \mathbf{G}_0 \sum_{n=1}^{n=\infty} \left(\sum_{k=1}^{k=\infty} \mathbf{V}_k \right)^n \mathbf{G}_0^n \tag{5}$$

so that

$$\begin{aligned}
\mathbf{L} - \mathbf{L}_0 &= \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0 \\
0 &= \mathbf{G}_0\mathbf{V}_2\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0 \\
0 &= \mathbf{G}_0\mathbf{V}_3\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_2\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}_2\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0 \\
&\vdots \\
0 &= \mathbf{G}_0\mathbf{V}_n\mathbf{G}_0 + \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_{n-1}\mathbf{G}_0 \cdots + \mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0 \cdots \mathbf{V}_1\mathbf{G}_0 \\
&\vdots
\end{aligned}$$

Solving for $\mathbf{V}_k$

From which it follows that

$$
\begin{aligned}
\mathbf{V}_1 &= \mathbf{L}_0(\mathbf{G}-\mathbf{G}_0)\mathbf{L}_0 \\
\mathbf{V}_2 &= -\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
\mathbf{V}_3 &= -\mathbf{V}_1\mathbf{G}_0\mathbf{V}_2 - \mathbf{V}_2\mathbf{G}_0\mathbf{V}_1 - \mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
&= +\mathbf{V}_1\mathbf{G}_0(\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1) + (\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1)\mathbf{G}_0\mathbf{V}_1 - \mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
&= +\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
\mathbf{V}_4 &= -2\mathbf{V}_1\mathbf{G}_0\mathbf{V}_3 - \mathbf{V}_2\mathbf{G}_0\mathbf{V}_2 - 3\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}\mathbf{V}_2 - \mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
&= -\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
\mathbf{V}_5 &= -2\mathbf{V}_1\mathbf{G}_0\mathbf{V}_4 - 2\mathbf{V}_2\mathbf{G}_0\mathbf{V}_3 - 3\mathbf{V}_1\mathbf{G}_0\mathbf{V}_2\mathbf{G}_0\mathbf{V}_2 \\
&\quad -3\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_3 - 4\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_2 - \mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
&= \mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1\mathbf{G}_0\mathbf{V}_1 \\
&\quad \vdots
\end{aligned}
$$

Solving for V_k

Which can be expressed in the simplified form

$$
\begin{aligned}
\mathbf{V}_1 &= \mathbf{L}_0(\mathbf{G}-\mathbf{G}_0)\mathbf{L}_0 \\
\mathbf{V}_2 &= (-1)^1(\mathbf{V}_1\mathbf{G}_0)^2\mathbf{L}_0 = (-1)^1(\mathbf{L}_0(\mathbf{G}-\mathbf{G}_0))^2\mathbf{L}_0 \\
\mathbf{V}_3 &= (-1)^2(\mathbf{V}_1\mathbf{G}_0)^3\mathbf{L}_0 = (-1)^2(\mathbf{L}_0(\mathbf{G}-\mathbf{G}_0))^3\mathbf{L}_0 \\
\mathbf{V}_4 &= (-1)^3(\mathbf{V}_1\mathbf{G}_0)^4\mathbf{L}_0 = (-1)^3(\mathbf{L}_0(\mathbf{G}-\mathbf{G}_0))^4\mathbf{L}_0 \\
\mathbf{V}_5 &= (-1)^4(\mathbf{V}_1\mathbf{G}_0)^5\mathbf{L}_0 = (-1)^4(\mathbf{L}_0(\mathbf{G}-\mathbf{G}_0))^5\mathbf{L}_0 \\
&\vdots \\
\mathbf{V}_n &= (-1)^{n-1}(\mathbf{V}_1\mathbf{G}_0)^n\mathbf{L}_0) = (-1)^{n-1}(\mathbf{L}_0(\mathbf{G}-\mathbf{G}_0))^n\mathbf{L}_0 \\
&\vdots
\end{aligned}
$$

Solving for V_k

The inverse series can be written in the form

$$\mathbf{L} - \mathbf{L}_0 = \left[\mathbf{L}_0(\mathbf{G} - \mathbf{G}_0) - \sum_{n=2}^{\infty} (-1)^{n-1} \mathbf{L}_0^n (\mathbf{G} - \mathbf{G}_0)^n \right] \mathbf{L}_0 \tag{6}$$

and

$$\mathbf{G} - \mathbf{G}_0 = \left[\mathbf{G}_0(\mathbf{L} - \mathbf{L}_0) - \sum_{n=2}^{\infty} (-1)^{n-1} \mathbf{G}_0^n ((\mathbf{L} - \mathbf{L}_0))^n \right] \mathbf{G}_0 \tag{7}$$

Once again, we recognize a strong dependence on residuals. In the first the residual is based on what we would call data. In second it is between sources. Since the seismic source is usually unknown, the second is not likely to be considered part of any formal scattering based estimation scheme. However, whenever a reasonable estimate of a source is available it should be possible to base some form of inversion on the latter of the preceding equations.

Convergence of the forward or inverse scattering series is not really an issue for us, but for any given source ϕ or data U, the series converge whenever

$$\| \mathbf{L}_0 \left(\mathbf{G} - \mathbf{G}_0 \right) (\phi) \| < 1 \tag{8}$$

or

$$\| \mathbf{G}_0 \left(\mathbf{L} - \mathbf{L}_0 \right) (U) \| < 1. \tag{9}$$

These observations do not necessarily imply uniform convergence and, in fact, the rate of convergence can be source and data dependent. These two formulas simply provide a measure of how close the two residuals must be in order to guarantee convergence. What (8) and (9) imply is that finding a suitable **L** or **G** is highly dependent on how close the estimated is to the measured.

Free Surface Elimination

The first step in ISS inversion eliminates free-surface multiples from the measured data. This is accomplished by assuming that $\mathbf{G}_0 = \mathbf{G}_0^f + \mathbf{G}_0^d$ is expressible as a sum of a Green's function, $\mathbf{G}^f$, responsible for the wavefield due to the free-surface and a Green's function, $\mathbf{G}^d$, responsible for the direct propagating wavefield. One then computes the deghosted and first arrival eliminated data

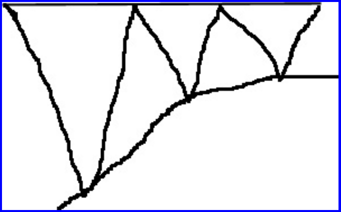
$$D_0' = \mathbf{G}_0 \left(\mathbf{L} - \mathbf{L}_0^f - \mathbf{L}_0^d \right) \mathbf{G}_0(\phi)$$

The multiple for each order given by

$$D_n' = D_0' D_n'$$

and the demultipled data is

$$D' = D_0' + \sum_{n=1}^{n=\infty} \phi^{-n} D_n'$$

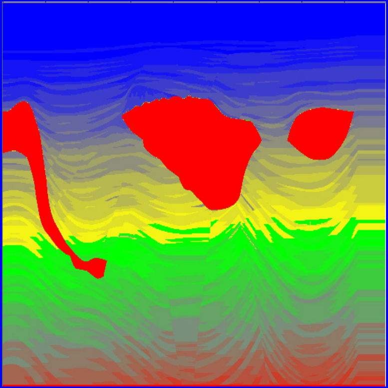


Second Order Multiple

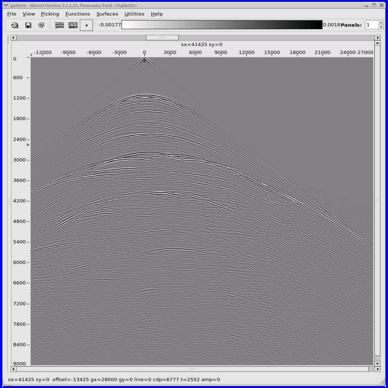
Practical Issues

- Once deghosting and first arrival elimination has been completed the multiple orders are computed directly from the inverse scattering series
- In practice the seismic wavelet ϕ is not known
  - Consequently subtraction of $\mathbf{G}_0 \left(\mathbf{L}_0^f - \mathbf{L}_0^d\right) \mathbf{G}_0$ may be an issue
  - Muting can be used to remove first arrival without knowing the wavelet
  - Removing ghosts is potentially a more complex issue
- The estimation of ϕ is incorporated into the multiple elimination stage
  - Thus ϕ is estimated as the wavelet minimizing $\| D_0' - \sum_{n=1}^{n=\infty} \phi^{-n} D_n' \|$
  - A more general approach minimizes $\| D_0' - \sum_{n=1}^{n=N} \phi_n^{-1} D_n' \|$
    - Which produces a new independent wavelet for each multiple order

Multiple Suppression at Pluto



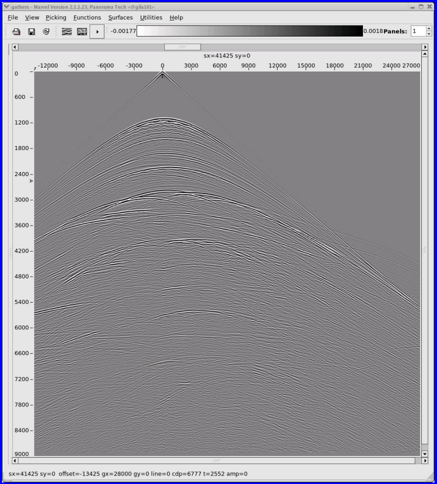
(a) Pluto Model



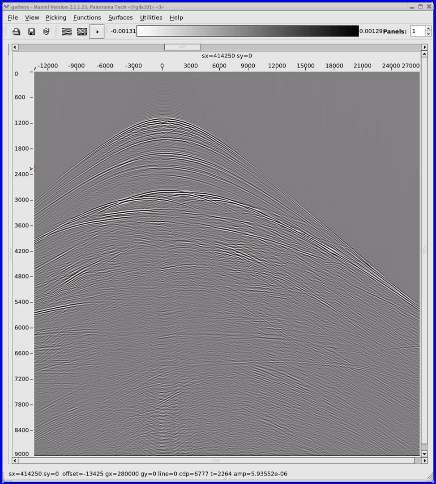
(b) Pluto Shot

A synthetic model (Pluto from SMAART JV) in (a) and a synthetic shot in (b)

Multiple Suppression at Pluto



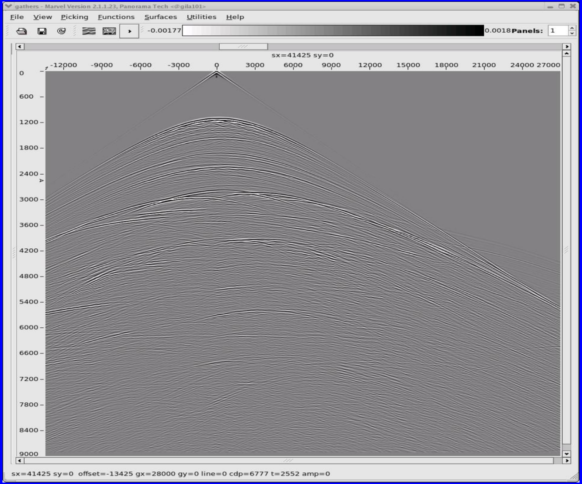
(c) Pluto Shot



(d) Pluto Muted Shot

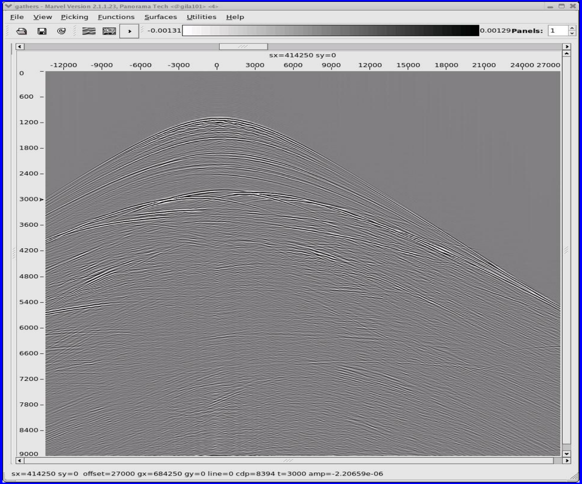
Synthetic Shot in (a) with muted and deghosted shot in (b).

Multiple Suppression at Pluto



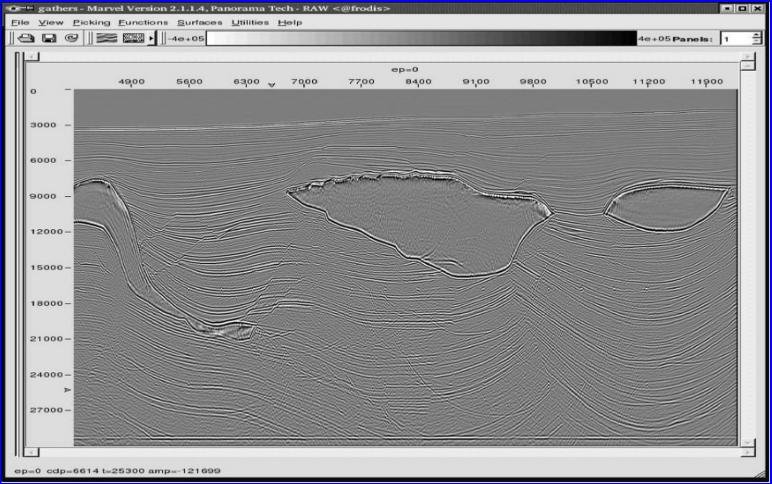
Raw shot record, no SRME

Multiple Suppression at Pluto



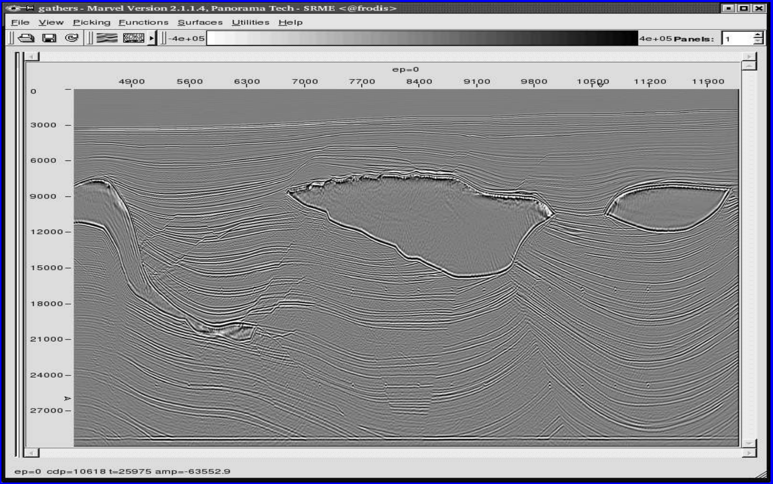
Raw shot record after SRME

Multiple Suppression at Pluto



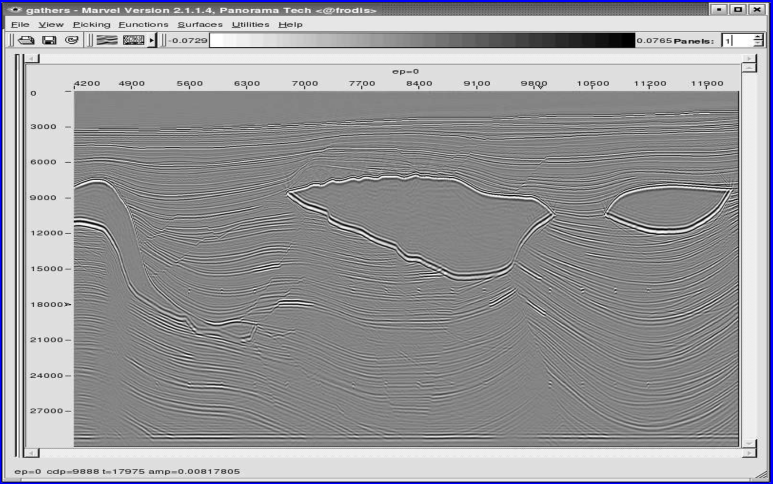
One-way prestack migration without SRME

Multiple Suppression at Pluto



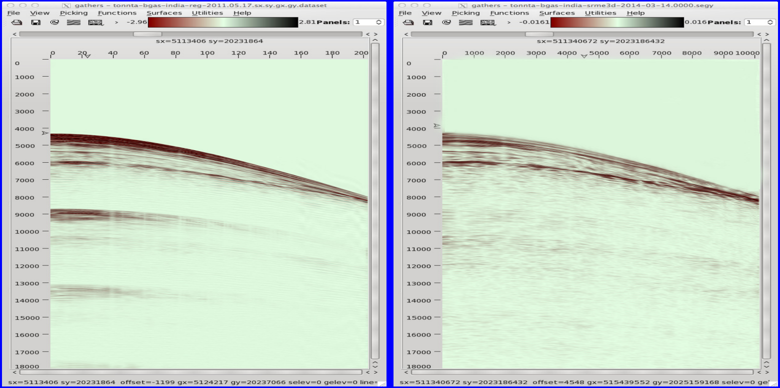
One-way prestack migration after SRME

Multiple Suppression at Pluto



Two-way (RTM) prestack migration after SRME

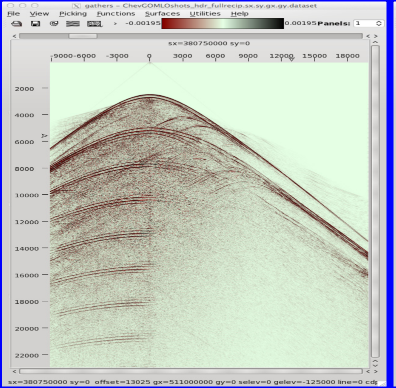
Complex multiple suppression



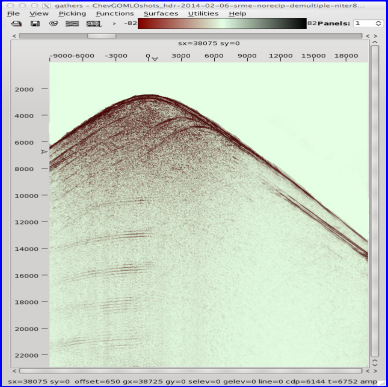
(a) Raw   (b) SRME

Part (a) shows a slice though raw shot record SRME. Part (b) is the record after SRME. This is eight order multiple elimination with a different wavelet for each order. Multiple suppression is quite good. The water bottom below this shot is smooth and Reflection strength below it is moderate.

Complex multiple suppression



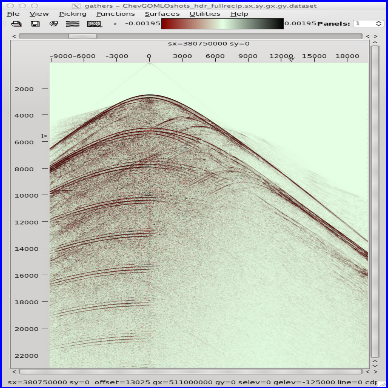
(a) Raw



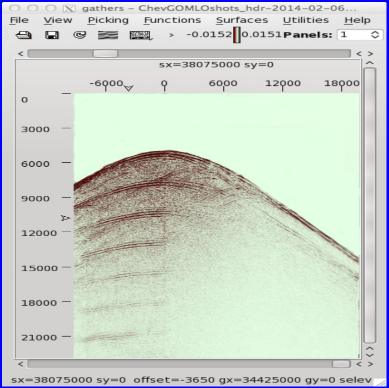
(b) SRME

Part (a) shows a slice though raw shot record. Part (b) is the record after SRME. This is eight order multiple elimination with a different wavelet for each order. Clearly, multiple suppression is not very good. An event directly below the water bottom has strong reflectivity inducing strong internal multiples.

Complex multiple suppression



(c) Raw

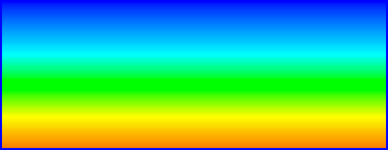


(d) Multiples

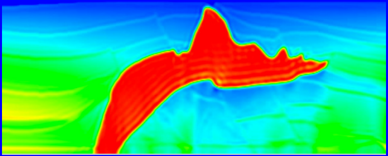
Part (a) shows a raw shot record. Part (b) shows the predicted multiples. It is not clear why the results in the previous slide are so bad.

FWI after SRME?

- Primaries and internal multiples
- No free surface multiples in ”data”
- Easy to synthesize such data
- Can it reduce FWI complexity?
- Figure at left seems to say yes



(a) Initial Model

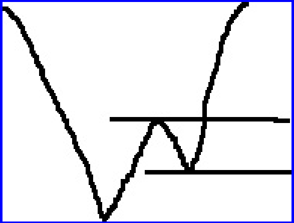


(b) FWI after SRME

Initial ($v(z)$) model (a). Inversion (b) based on demultipled data.

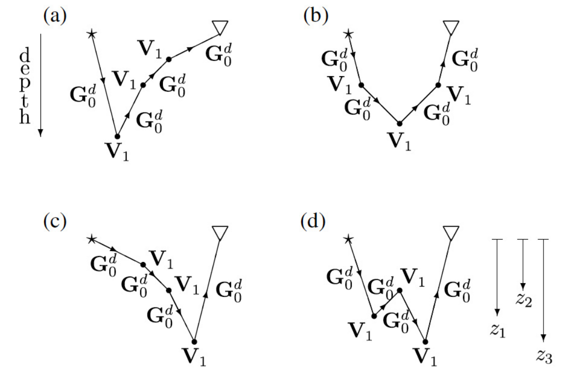
Second Step

- Internal multiple elimination
- Transform to a pseudo depth domain
  - Via constant velocity
- Pseudo depth-by-depth series series sum
- Amplitudes and arrival times not exact
- Not an exact or direct process
- Resulting data contains primaries only
- Improved one-way FWI scheme
  - SPEED



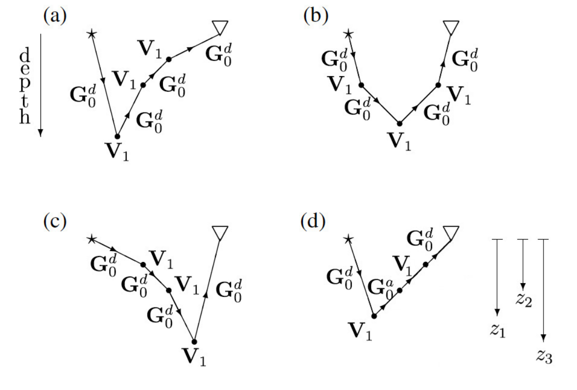
Internal Multiple

Internal Multiple Mechanism



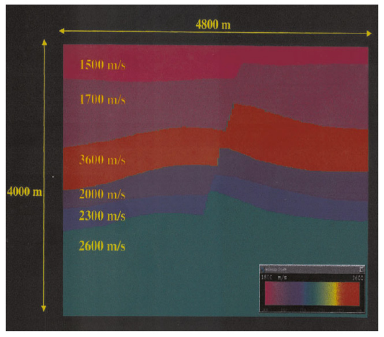
- Internal multiple mechanism in terms of V_1
  - Different parts of $\mathbf{G}_0^d V_1 \mathbf{G}_0^d V_1 \mathbf{G}_0 V_1 \mathbf{G}_0^d$
- Only odd terms of the inverse series
- Only (d) contributes to the first order internal multiple

Internal Multiple Mechanism



- Internal multiple mechanism in terms of V_1
  - Different parts of $\mathbf{G}_0^d V_1 \mathbf{G}_0^d V_1 \mathbf{G}_0 V_1 \mathbf{G}_0^d$
- Only odd terms of the inverse series
- In internal suppression, z_1 and z_2 are important

Synthetic Internal Multiple Suppression Example



(c) Model

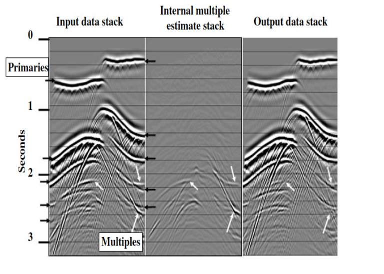


Figure 17. A 2D synthetic model (top). The left panel (bottom) shows a common offset display from the synthetic data set created using the model. The middle panel (bottom) shows the predicted internal multiples and the right-hand panel (bottom) is the result after subtracting the predicted multiples from the input data set.

(d) Internal Multiples and Stack

Simple velocity model (a). Stack, internal multiples, and stack without internal multiples (b).

Real Data Internal Multiple Suppression Example

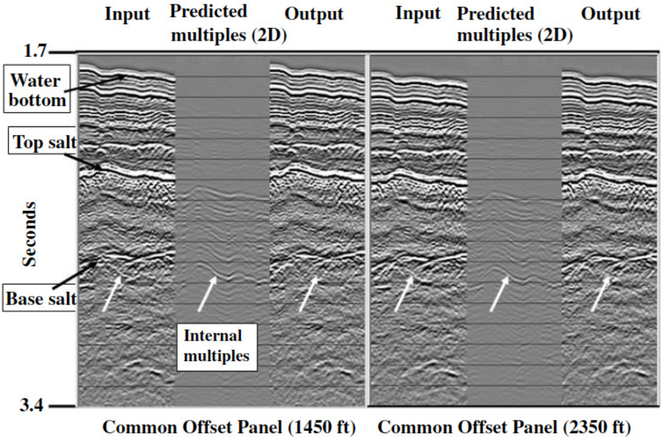


Figure 20. An example of inverse-scattering internal multiple attenuation from the Gulf of Mexico. Data are courtesy of WesternGeco.

- In theory, the final ISS step produces a precise depth image
  - By summing a series using the free-surface and internal multiple suppressed data set
  - To date no reasonable example has been forthcoming
  - I have made no attempt to perform this final step and have no further comments regarding it.

Outline

1. **Fundamental Principles**
2. **Green's Functions, Operators, Data and Sources**
   - Functions
   - Operators
   - Data and Sources
3. **Full Waveform Inversion**
   - Operator Form
4. **Inverse Scattering**
   - The Lippmann-Schwinger Equation
   - The Forward Scattering Series
   - The Inverse Series
   - Convergence
   - Multiple Elimination
   - Internal Multiple Elimination
   - The Final Step
5. **Final Comments**

- Both ISS and FWI are valid approaches to the inversion of seismic data
- Combining the two methodologies may provide new and interesting results
- Both require similar acquisitions to be successful
- Both are inherently based on manipulation of some form of residual
- Removal of free-surface multiples is a significant advantage for FWI.
- Data without free-surface and internal multiples is suitable for one-way inversion

Questions?