Seismic Modeling, Migration and Velocity Inversion

Full Waveform Inversion

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Outline

Outline

AVA Based "Inversion"

Prestack *inversion* is sometimes based on minimizing

$$
\begin{aligned}
F(I_P, I_S, \rho) &= \alpha \sum_{i,j} \left[S_{ij}^{data} - S_{ij}(I_P, I_S, \rho) \right]^2 + \sum_{ij} \| R_{ij}(I_P, I_S, \rho) \| \\
&+ \sum_{j} \left[\| I_P^{low} - \hat{I}_P^{low} \| + \| I_S^{low} - \hat{I}_S^{low} \| + \| \rho^{low} - \hat{\rho}^{low} \| \right]
\end{aligned}
$$

subject to

$$
\begin{aligned}
I_{Pmin} &\leq I_P \leq I_{Pmax} \\
I_{Smin} &\leq I_S \leq I_{Smax} \\
\rho_{min} &\leq \rho \leq \rho_{max}
\end{aligned}
\tag{1}
$$

AVA Based "Inversion"

Notes:

- I_P, I_S, are P and S impedance and ρ is density in time
- i is the angle-stack index
- j is the sample index
- $R_{ij}(I_P, I_S, \rho)$ is the angle-dependent PP reflectivity
- S_{ij}^{data} is the measured *AVA* amplitude
- $S_{ij}(I_P, I_S, \rho)$ is the 1D numerically simulated synthetic seismic data

Variables with *low* superscripts designate low-frequency components of P-impedance, S-impedance, and density respectively.

AVA Based ”Inversion”

This is not Full Waveform Inversion

Outline

Full Waveform Inversion

- For a given model
  - For each observed shot, synthesize data to match the real acquisition
    - Use a full two-way modeling algorithm
    - Save a trace at each model node
  - Compute the difference between the shot and the real data
    - These data are called the residuals
  - Back propagated the residuals into the model
    - Use a full two-way modeling algorithm
    - Save a trace at each model node
  - Preform a shot-profile migration of the residuals
    - The shot is the forward-propagated synthetic
    - The receiver traces are the back-propagated residuals
    - Divide the back by the forward propagated traces
  - Normalize the image above by the velocity squared
  - Add the normalized image to the current model
  - Repeat the previous steps until the norm of the model difference is small
- FWI is really a iterative migration scheme

Outline

Variational Formulation of the Wave Equation

The two-way-frequency-domain-scalar wave equation

$$-\frac{\omega^2}{c^2} - \nabla^2 U = f(\vec{x}_s, \omega), \tag{2}$$

where $f(\vec{x}_s, \omega)$ is a compressional source located at $\vec{x}_s$, and $c(\vec{x})$ is velocity, has the variational form

$$\phi(U, V) = -\int_\Omega \frac{\omega}{c^2} V d\Omega + \int_\Omega \nabla U \nabla V d\Omega = f(\vec{x}_s, t) \tag{3}$$

where V are functions used to approximate $U(x, y, z)$.

The Matrix Form

Given a family, V_k, of approximating functions s we can approximate U, and f by $u(\vec{x}) = \sum_{k=1}^{n} U_k V_k(\vec{x})$ and $f(\vec{x}_s, \omega) = \sum_{k=1}^{n} f_k V_k(\vec{x})$ so that the variational form in equation (3)

$$\sum_{k=1}^{n} U_k \phi(V_k, V_j) = \sum_{k=1}^{n} f_k \int_{\Omega} V_k V_j d\Omega \tag{4}$$

can be expressed in matrix form as

$$\mathbf{S}\vec{U} = \mathbf{M}\vec{f} \tag{5}$$

Here **S** is called the complex impedance matrix and **M** the stiffness matrix.

Notes

- $\mathbf{S}\vec{U} = \mathbf{M}\vec{f}$ is a single frequency equation.
- The matrix **M** does not depend on U_k. Its more like a new source term.
- With proper choice of $\{V_k\}$ we can arrange for $\mathbf{M} = I$.
- For our purposes here and to simplify notation,

$$\mathbf{S}\vec{U} = \vec{f} \tag{6}$$

- We assume that **S** is square, symmetric, and invertible.
- The inverse, $\mathbf{S}^{-1}$, of **S** is a modeling "operator"
- Thus

$$\vec{U} = \mathbf{S}^{-1}\vec{f} \tag{7}$$

Full Waveform Inversion

Full waveform inversion begins with a suitably chosen objective function which for the classical case is

$$E = \| J(\vec{D}, \vec{U}) \| = \| \vec{D} - \vec{U} \| \,. \tag{8}$$

where $\| \cdot \|$ is the usual least squares norm, $\vec{D}$ is the observed seismic data and $\vec{U} = \mathbf{S}^{-1}\vec{f}$ is synthetic data corresponding to the current velocity model estimate.

The Inversion Scheme

Given an initial velocity model, we can consider two update schemes:

- Move in the negative direction of the gradient of E.
- Use the full Newton method (Lines and Treitel 1984) to update the current model.

Choosing the second means that our updating scheme immediately takes the form

$$\vec{c}^{\;n} = \vec{c}^{\;n-1} - \mathbf{H}^{-1} \nabla_{\vec{c}^{\;n-1}} E \tag{9}$$

Thus, we must calculated the gradient of E and also invert the Hessian matrix **H**.

The Inversion Scheme cont'd

Finding the gradient of the objective function E requires that we find the gradient of $\vec{U} = \mathbf{S}^{-1}\vec{f}$ with respect to the sampled velocity model $\{c_k\}$ Thus,

$$\frac{\partial \mathbf{S}}{\partial c_k}\vec{U} + \mathbf{S}\frac{\partial \vec{U}}{\partial c_k} = 0 \tag{10}$$

$$\frac{\partial \vec{U}}{\partial c_k} = \mathbf{S}^{-1}\vec{P}_k \tag{11}$$

and

$$\vec{P}_k = -\frac{\partial \mathbf{S}}{\partial c_k}\vec{U} \tag{12}$$

where the middle equation defines what is normally called the partial derivative wave field, and the bottom equation defines the so called virtual source vector required to perturb k-th velocity element.

The Inversion Scheme cont'd

From the objective function

$$\frac{\partial E}{\partial c_k} = \mathbf{Re}\left\{ \begin{bmatrix} \frac{\partial U_1}{\partial c_k} & \frac{\partial U_2}{\partial c_k} & \cdots & \frac{\partial U_{nr}}{\partial c_k} & \cdots & \frac{\partial U_{nn}}{\partial c_k} \end{bmatrix} \begin{bmatrix} (\widehat{U_1 - D_1}) \\ (\widehat{U_2 - D_2}) \\ \vdots \\ (\widehat{U_{nr} - D_{nr}}) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right\} \tag{13}$$

and from the partial derivative wave field

$$\frac{\partial E}{\partial c_k} = \mathbf{Re}\left\{ (\vec{P}_k)^T \mathbf{S}^{-1} \vec{r} \right\} \tag{14}$$

where

$$\vec{r} = \left[(\widehat{U_1 - D_1}), (\widehat{U_2 - D_2}), \cdots (\widehat{U_{nr} - D_{nr}}), 0, \cdots, 0 \right]^T \tag{15}$$

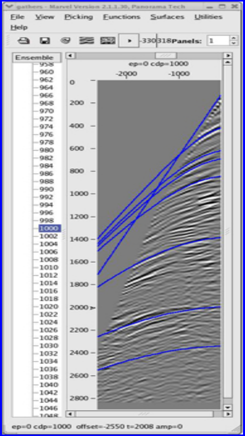
The Inversion Scheme cont'd

Finally, we approximate the Hessian via $\vec{P}_k$ so that for each k, the updating scheme is

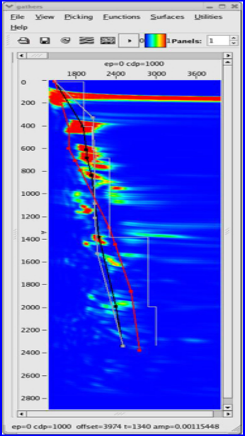
$$c_k^{l+1} = c_k^l + \alpha \sum_{\omega} \frac{\mathbf{Re}\left\{(\vec{P}_k)^T \mathbf{S}^{-1} \vec{r}\right\}}{\mathbf{Re}\left\{(\vec{P}_k)^T \widehat{\vec{P}_k} + \lambda\right\}} \tag{16}$$

Outline

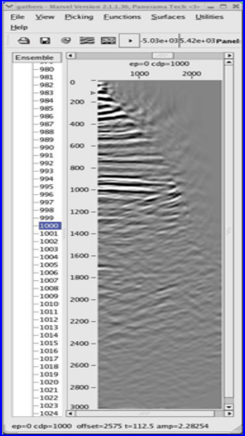
Marmousi MVA



(a) Gather Picks



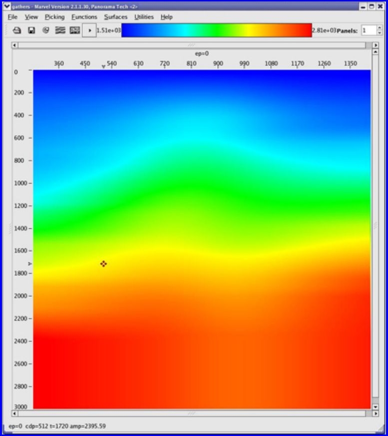
(b) Semblance Picks



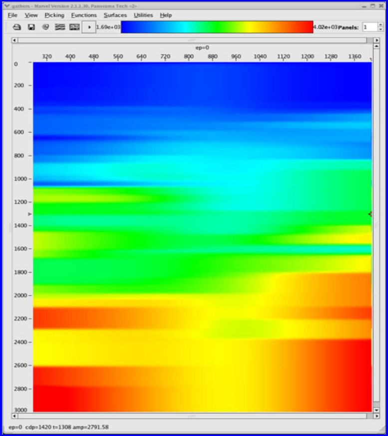
(c) NMO'd Gather

Typical Marmousi gather with picks, a semblance panel with picks, and the NMO corrected gather.

Marmousi MVA



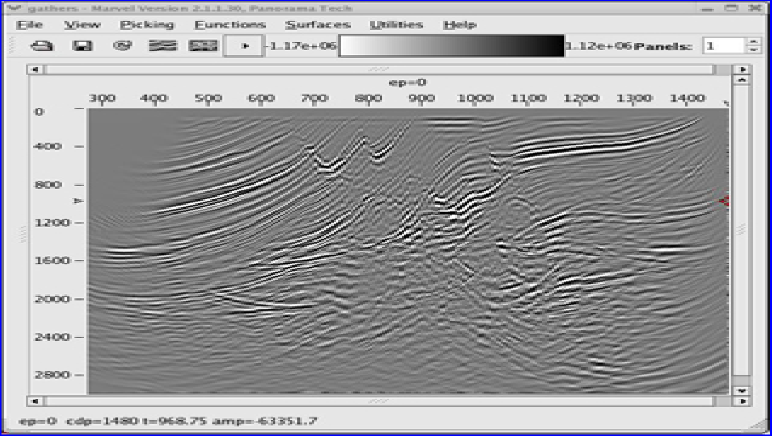
(d) Marmousi Time-RMS model



(e) Marmousi Depth-Interval model

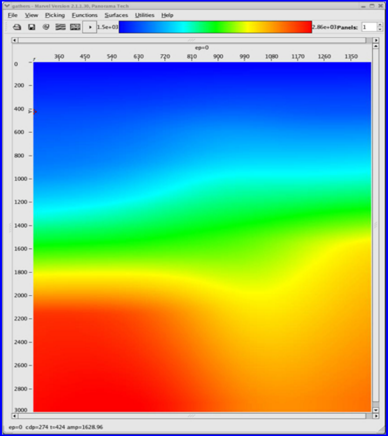
Initial stacking velocity models in time-RMS (left) and interval-depth (right).

Marmousi MVA

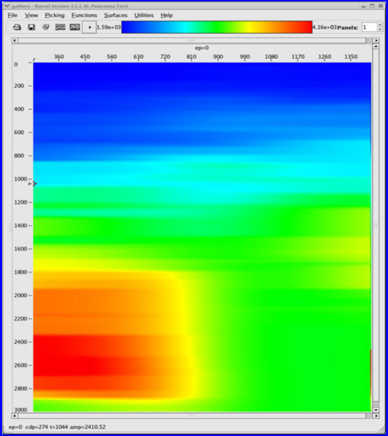


First iteration Marmousi stacking velocity based Kirchhoff migration.

Marmousi MVA



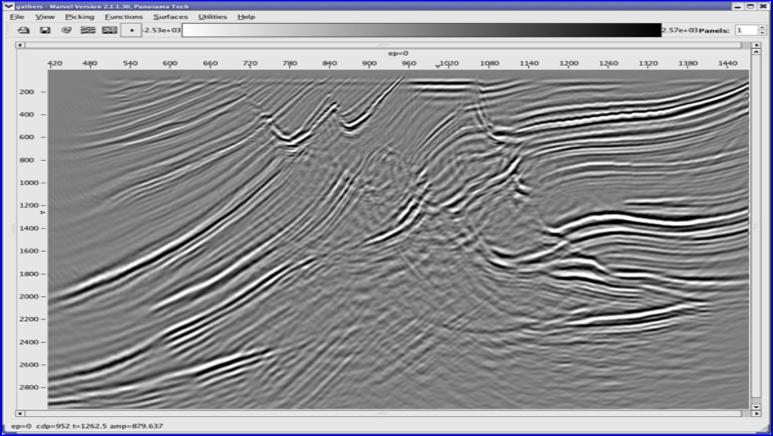
(f) Marmousi Time-RMS model



(g) Marmousi Depth-Interval model

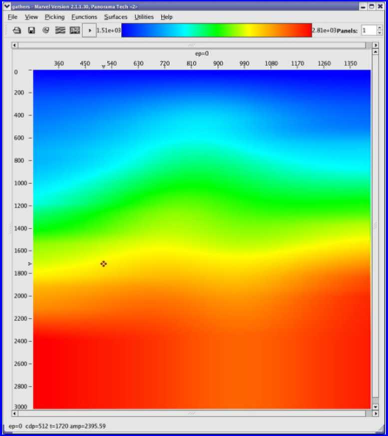
Second Kirchhoff based MVA models in time-RMS (left) and interval-depth (right).

Marmousi MVA

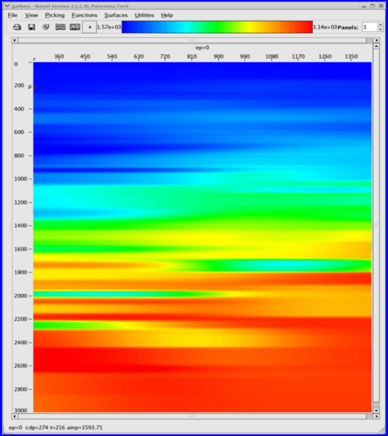


Second iteration Marmousi Kirchhoff based MVA Kirchhoff migration.

Marmousi MVA



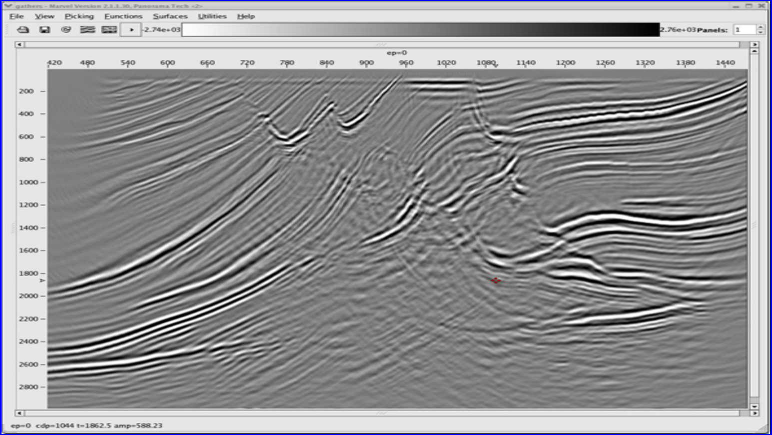
(h) Marmousi Time-RMS model



(i) Marmousi Depth-Interval model

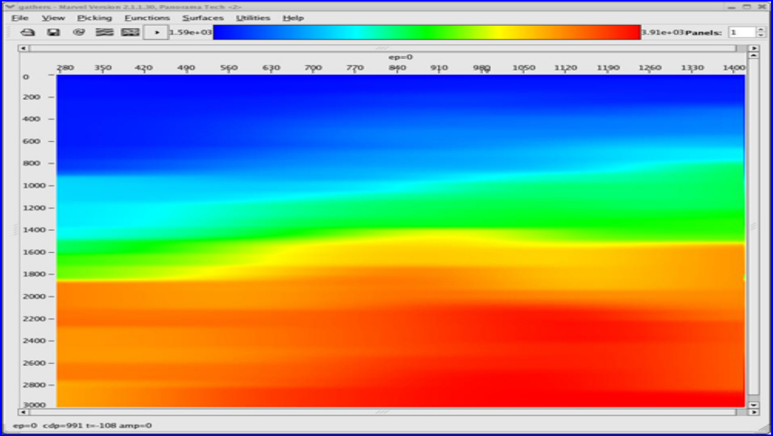
Second Kirchhoff based MVA models in time-RMS (left) and interval-depth (right).

Marmousi MVA



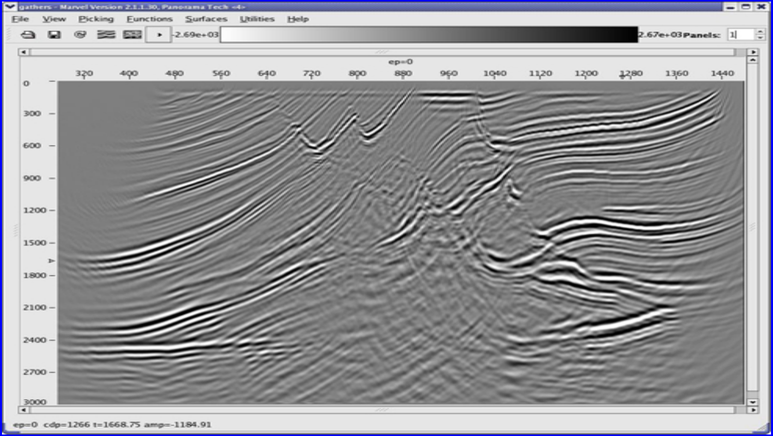
Third iteration Kirchhoff based MVA Kirchhoff migration.

Marmousi MVA



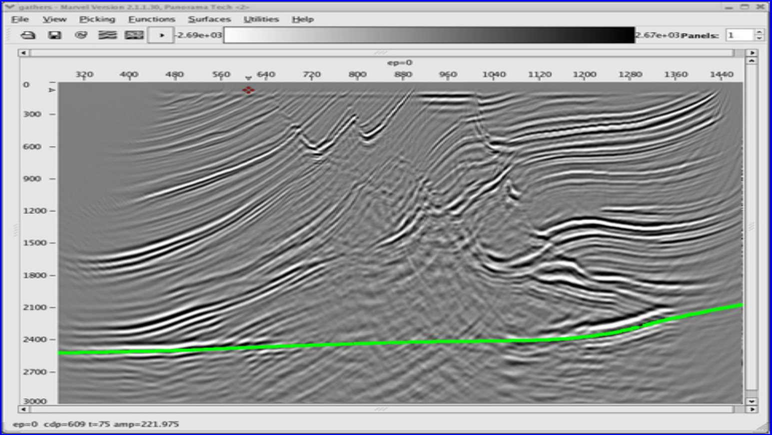
Fourth iteration Kirchhoff MVA based velocity model.

Marmousi MVA



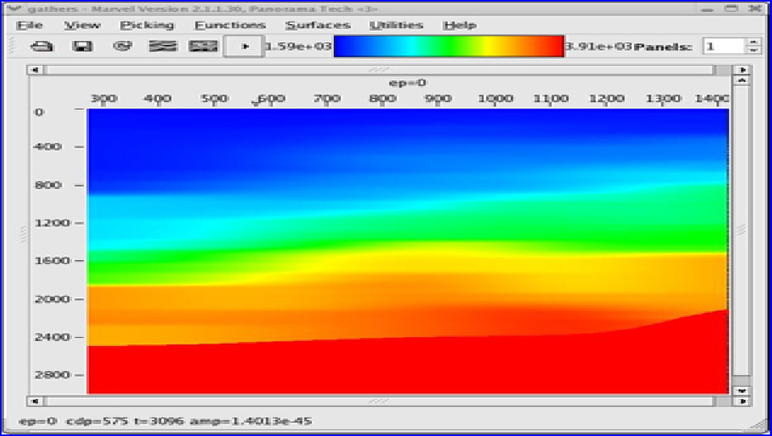
Fourth iteration Kirchhoff MVA based Kirchhoff migration.

Marmousi MVA



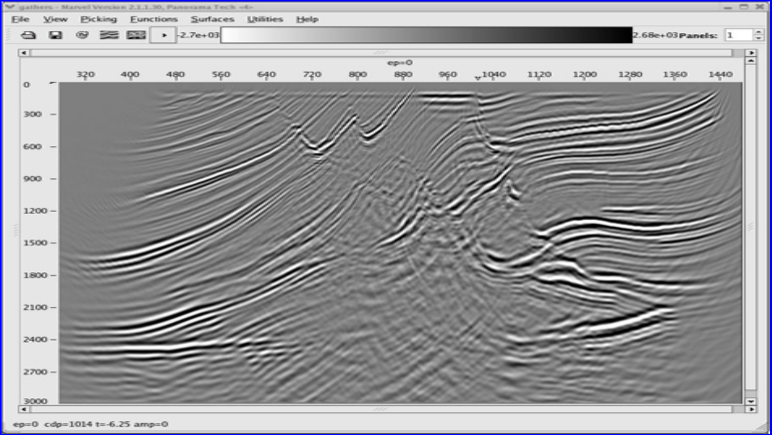
Bottom horizon for constant velocity analysis.

Marmousi MVA



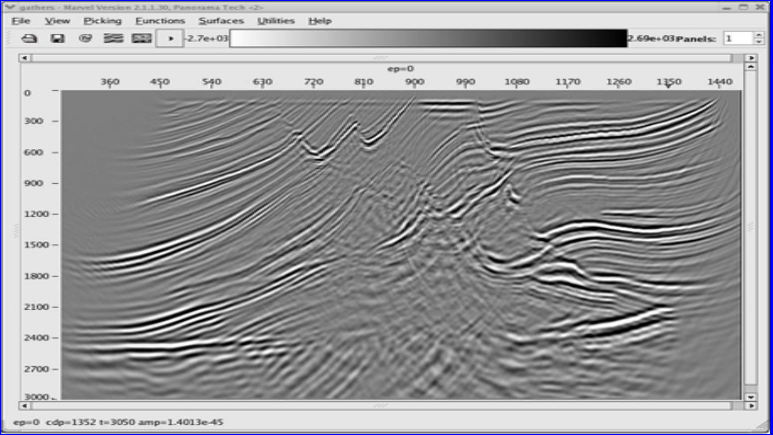
Fourth iteration Kirchhoff MVA based model with bottom horizon 4000 meter/second velocity flood.

Marmousi MVA



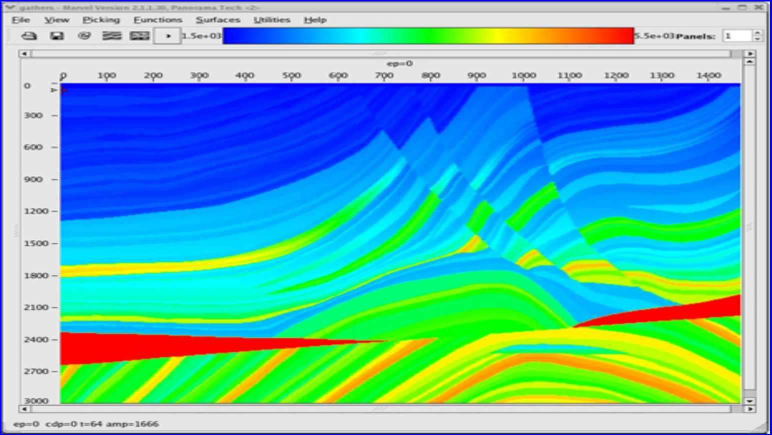
Fourth iteration Kirchhoff MVA based model with bottom horizon 4000 meter/second velocity flood migration.

Marmousi MVA



Fourth iteration Kirchhoff MVA based model with bottom horizon 5000 meter/second velocity flood migration.

Marmousi MVA

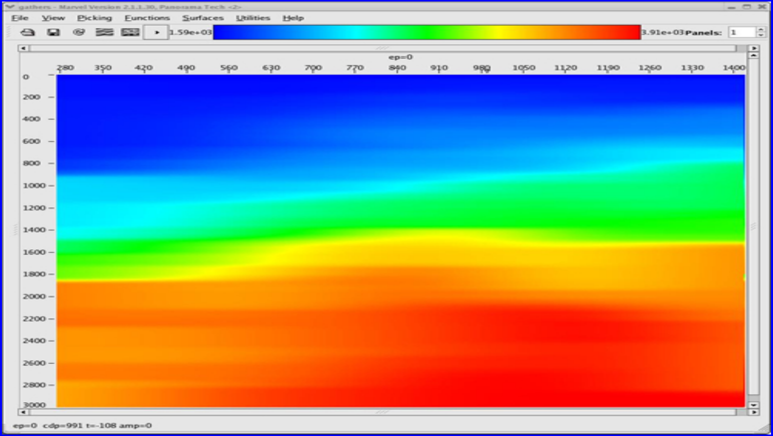


The true Marmousi model.

Notes

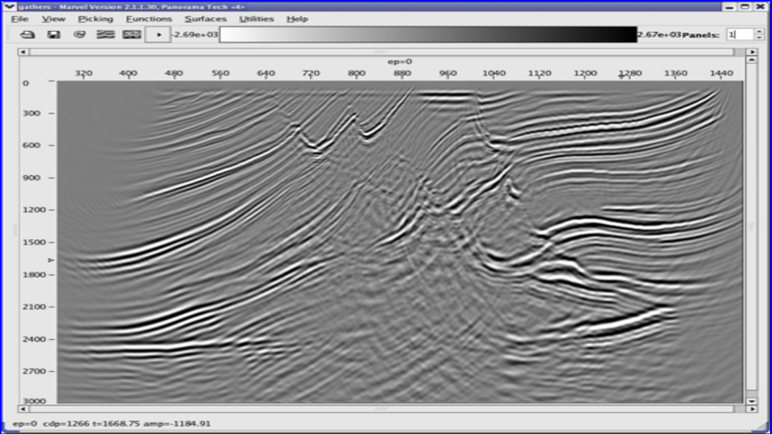
- Insufficient offset
  - Max of 2600 over 9000 km model
  - Approximately 1300 km velocity analysis basement
- Recording time too short (3 seconds)
- Long delay wavelet

Marmousi Inversion



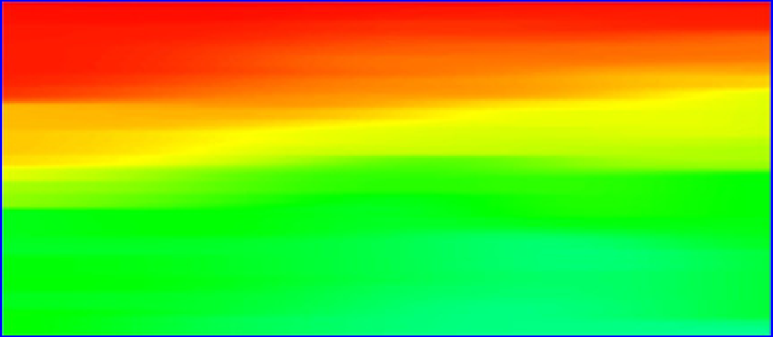
Estimated Marmousi velocity model from the iterative Migration Velocity Analysis above.

Marmousi MVA



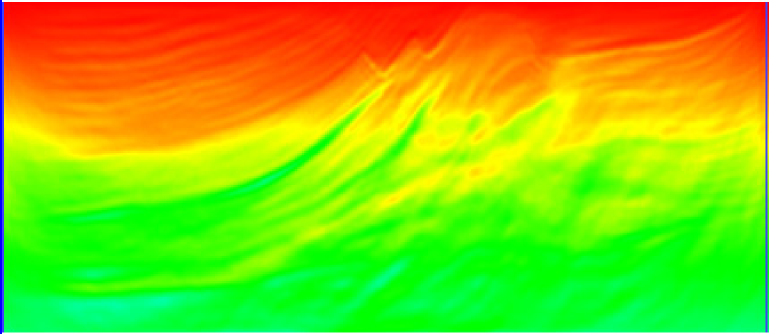
Kirchhoff migration using fourth iteration MVA.

Marmousi Inversion



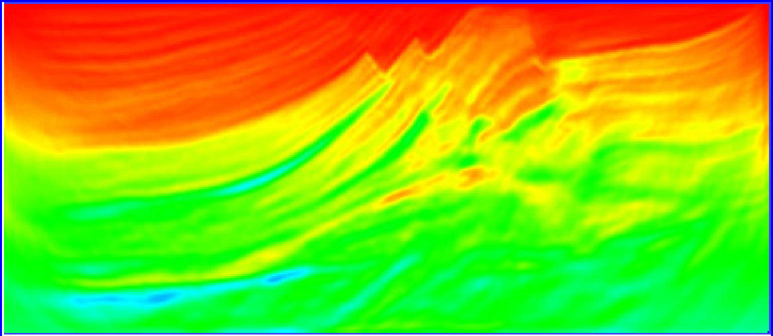
Estimated Marmousi velocity model. This model was obtained through iterative Migration Velocity Analysis.

Marmousi Inversion



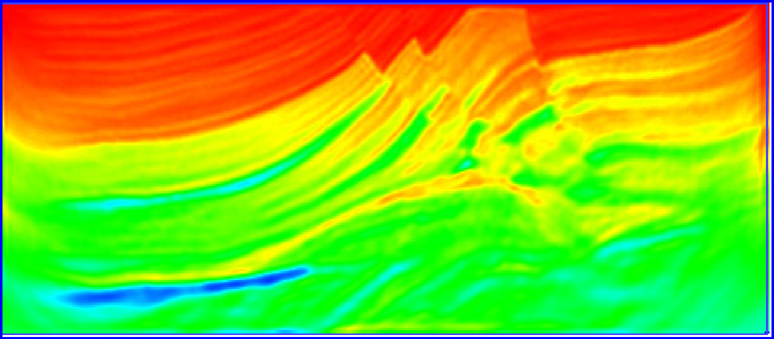
Estimated Marmousi velocity model after 6 iterations

Marmousi Inversion



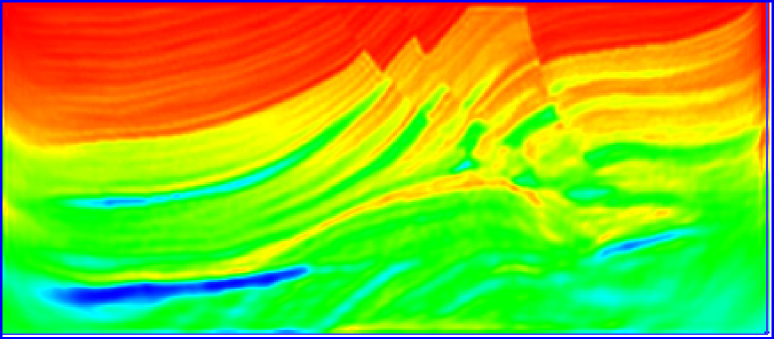
Estimated Marmousi velocity model after 12 iterations

Marmousi Inversion



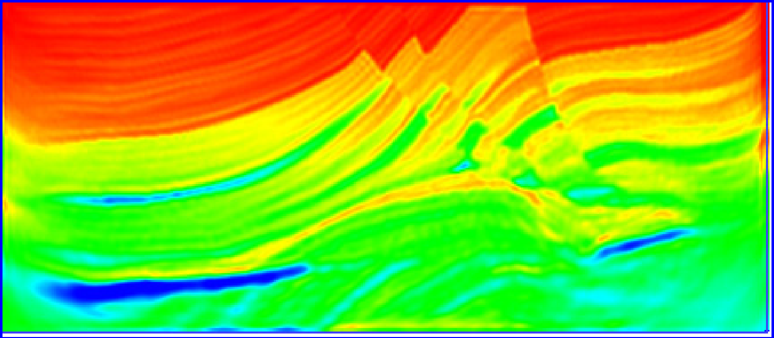
Estimated Marmousi velocity model after 18 iterations

Marmousi Inversion



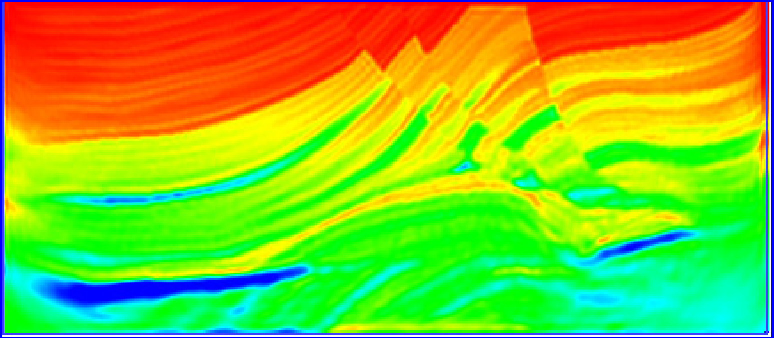
Estimated Marmousi velocity model after 24 iterations

Marmousi Inversion



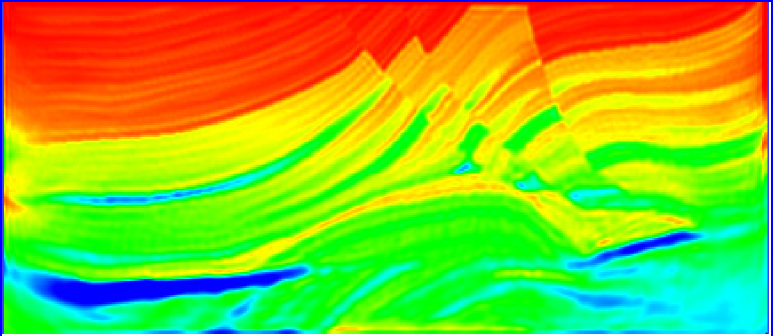
Estimated Marmousi velocity model after 30 iterations

Marmousi Inversion



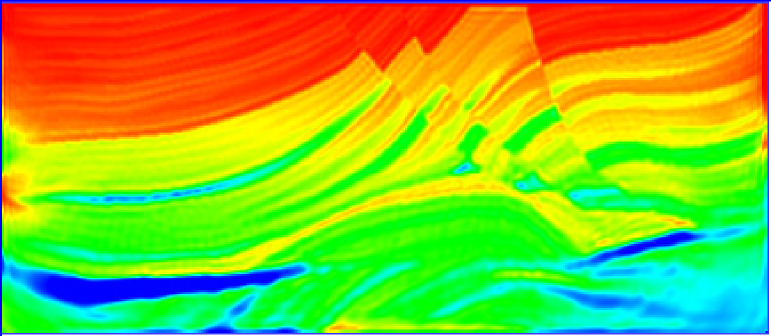
Estimated Marmousi velocity model after 42 iterations

Marmousi Inversion



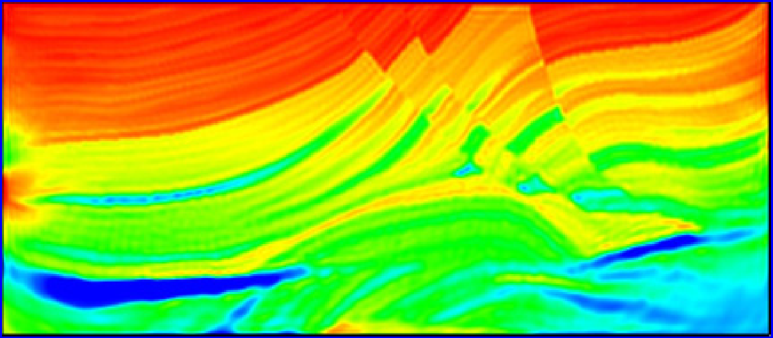
Estimated Marmousi velocity model after 48 iterations

Marmousi Inversion



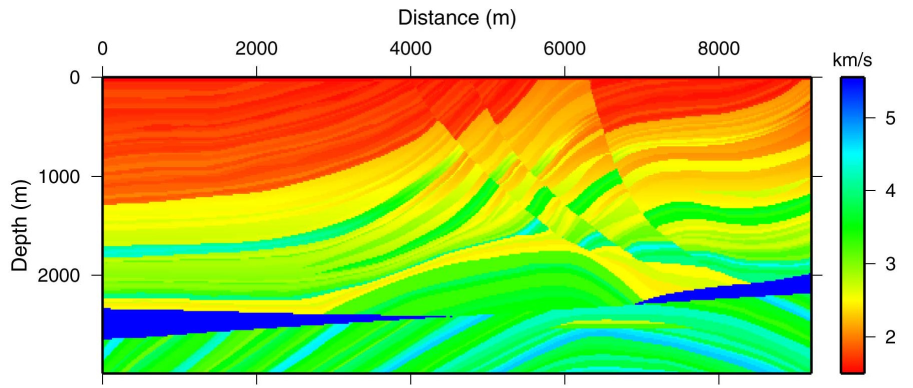
Estimated Marmousi velocity model after 54 iterations

Marmousi Inversion



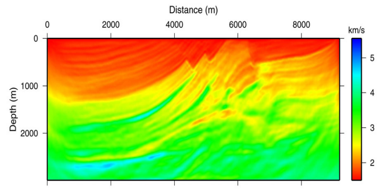
Estimated Marmousi velocity model after 60 iterations

Marmousi Inversion

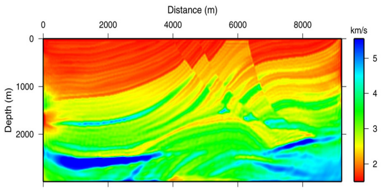


True Marmousi model.

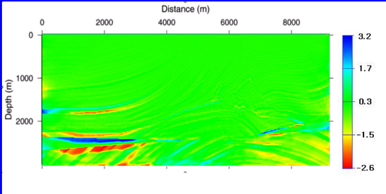
Marmousi Inversion



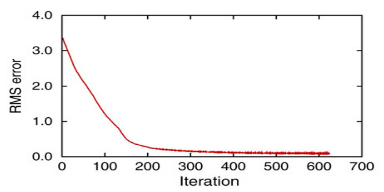
(j) After 100 iterations



(k) After 600+ iterations



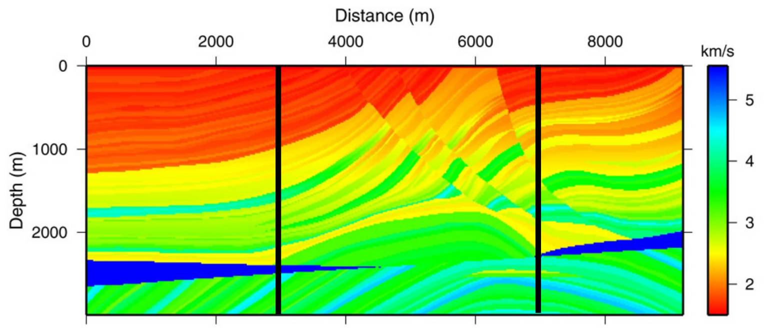
(l) Velocity error (600+ iterations)



(m) The RMS error

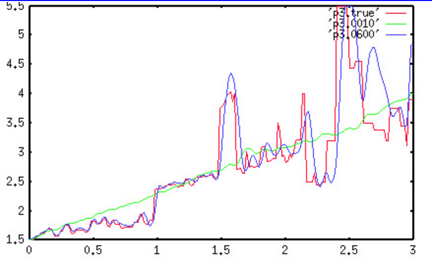
Marmousi Full Waveform Inversion

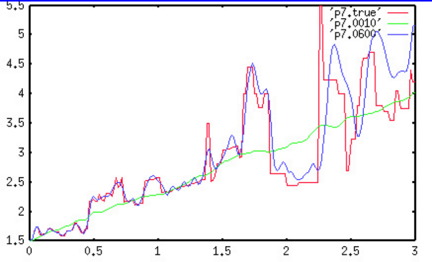
Marmousi Inversion



Log extraction locations.

Marmousi Inversion



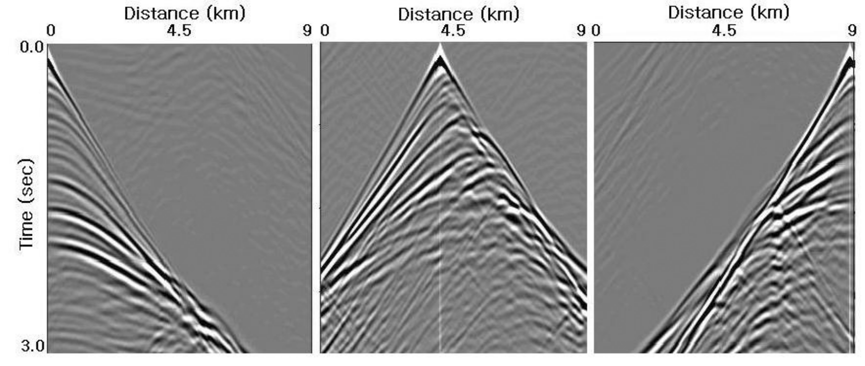


Inverted Versus True Logs at the locations specified in the previous slide.

Process Review

- The true model
  - Nine km by three km (depth)
- The *observed* data
  - Nine km offset
  - Broadband wavelet from .3 HZ to 50 HZ
    - Low frequency and long offsets are the key
  - Five second recording time
  - Model grid was 16m X 16m

The observed data



Marmousi Synthetic Data

The inversion process

- We started with a MVA model
  - Virtually no reflections
  - Reasonably accurate shallow
  - First iteration essentially muted the first breaks
  - First iteration is exactly equivalent to migrating with our initial model
    - Lailly: Migration is the first step in inversion
- We calculated a new velocity model from residuals and a synthetic shot
- We shot a new synthetic data set
- We imaged the residuals
- We repeated the exercise until model differences became negligible
- In this case the model is as good as can be expected

This kind of inversion is theoretically valid for all Earth Models.

Questions?