Seismic Modeling, Migration and Velocity Inversion

Non Finite Difference FKX Methods

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Outline

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Pseudo-Spectral for the Pressure Equation

Modeling with the pseudo-spectral method is best explained when based on the pressure equation

$$\frac{1}{\rho v^2}\frac{\partial^2 p}{\partial t^2} = \left[\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial p}{\partial z}\right].$$

The idea is to compute all but the time differential using Fast Fourier transforms. We start by approximating the left hand side

$$\frac{1}{\rho v^2}\frac{\partial^2 p}{\partial t^2} \approx \frac{1}{\rho v^2}\left(\frac{p(x,y,z,t+\Delta t) - 2p(x,y,z,t) + p(x,y,z,t-\Delta t)}{\Delta t^2}\right)$$

The Propagating Equation

The propagating equation for the pseudo-spectral method is then

$$
\begin{aligned}
p(x, y, z, t+\Delta t) &= 2p(x, y, z, t) - p(x, y, z, t-\Delta t) \\
&+ \rho (v \Delta t)^2 \left[\frac{\partial}{\partial x} \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \frac{1}{\rho} \frac{\partial p}{\partial z} \right].
\end{aligned}
$$

The idea is to compute the discrete version of the term in brackets using Fourier transforms.

The Pseudo Spectral Method

Calculation of

$$\left[\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial p}{\partial z}\right]$$

is achieved by calculating each of the partial derivatives

$$\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial p}{\partial x}, \frac{\partial}{\partial y}\frac{1}{\rho}\frac{\partial p}{\partial y}, \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{1}$$

in turn. Clearly, knowing how to do one provides the recipe for the others.

Computing the Partial Derivatives

Let $P = \frac{1}{\rho}\frac{\partial p}{\partial x}$ then

$$\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial P}{\partial x} \tag{2}$$

and the recipe for computing the term in brackets is

- Fourier transform
- Multiply by $i\omega$
- Inverse Fourier transform
- Multiple by $\frac{1}{\rho}$
- Fourier transform
- Multiply by $i\omega$
- Inverse Fourier transform
- Perform in turn for x,y, and z
- Update in time
- Repeat for the next time stamp

The Pseudo Spectral Method

- The advantages
  - No dip limits
  - Accurate amplitudes
  - Accurate phase
- The disadvantages are
  - Large memory requirements
    - Cluster required
  - Swapping reduces efficiency
- Equivalent to central differences with a huge number of coefficients.
- Lack of efficiency reduces popularity

Outline

The FK Equation

We will now focus on Fourier based methods for solutions of the one-way equation

$$\frac{\partial U}{\partial z} = \pm\sqrt{(k^2 - k_x^2 - k_y^2)}U \quad (3)$$

where $k = \frac{\omega}{v}$. Taylor series expansion of the square root term provides the method for solving this equation.

Taylor Series Approximation in FK

We begin by expanding

$$k_Z = \pm i\sqrt{k^2 - k_X^2 - k_Y^2}$$

in a Taylor series relative to a reference velocity $v_0(z)$. In terms of the slowness difference $\Delta s = \frac{1}{v} - \frac{1}{v_0}$ we can write

$$k_Z = \pm i\left(\sqrt{k_0^2 - k_X^2 - k_Y^2} + \omega\Delta s + \frac{2(k_X^2 + k_Y^2)}{4k_0^2 - 3(k_X^2 + k_Y^2)}\omega\Delta s^2 + \cdots\right)$$

where $k_0 = \frac{\omega}{v_0}$.

The First Term

Ignoring the second and third terms of the Taylor expansion yields the differential equation

$$\frac{\partial u}{\partial z} = \pm i k_z u.$$

with $k_z = \sqrt{k_0 - k_X^2 - k_Y^2}$. Within each constant velocity interval $[z, z + \Delta z]$ the solution of this equation is

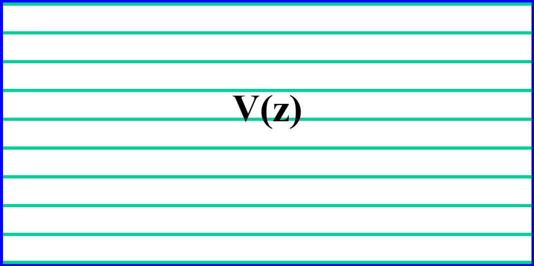
$$p(k_x, k_y, z + \Delta z, \omega) = \exp\left[\pm i k_z \Delta z\right] p(k_x, k_y, z, \omega)$$

Thus downward or upward continuation is seen simply as a multiplication by a complex exponential.

In this case we have assumed that the propagation is governed by a velocity $v_0(z)$ that has no lateral variation.

Phase Shift in FK

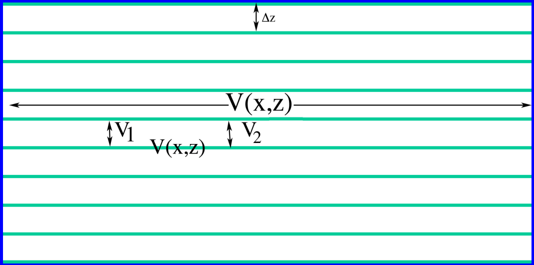
- Slice-by-slice
  - Each step in FK



FK domain depth-slice by depth-slice continuation for $v(z)$ velocity models. This is called phase shift modeling.

Phase Shift Plus Interpolation PSPI in FK

- Slice-by-slice
  - Several Velocities in FK
  - Interpolate



PSPI FK domain depth-slice by depth-slice continuation for $v(x, y, z)$ velocity models.

The First and Second Terms

Using the first two terms of the Taylor series splits the modeling into a step in FK:

$$p_1(k_x, k_y, z + \Delta z, \omega) = \exp\left[\pm i k_z \Delta z\right] p(k_x, k_y, z, \omega)$$

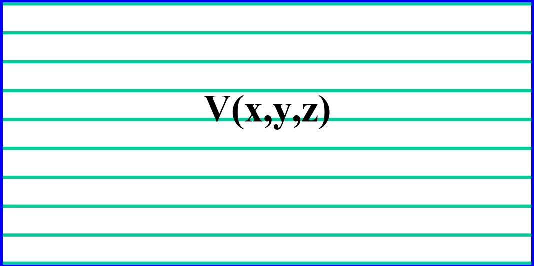
with $k_z = \sqrt{k_0 - k_X^2 - k_Y^2}$, Followed by a step in FX:

$$p(x, y, z + \Delta z, \omega) = \exp\left[\pm i \omega \Delta s \Delta z\right] p_1(x, y, z + \Delta z, \omega)$$

This is normally called split-step modeling.

Split Step in FKX

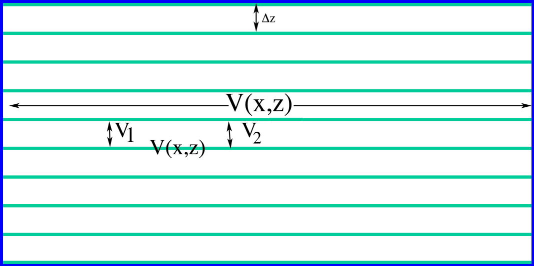
- Slice-by-slice
  - First step FK
  - Second step FX



Split-step in 3D.

Split Step Plus Interpolation in FKX

- Slice-by-slice
  - For several velocities
    - Split-Step in FK and FX
  - Interpolate for given velocity



Split-step plus interpolation in 3D.

All Three Terms

After applying the first two split-step terms our problem is to calculate f_z^{fd} in

$$p(k_x, k_y, z + \Delta z, \omega) = e^{ik_z^{fd}\Delta z} p_2(k_x, k_y, z + \Delta z, \omega)$$

Here, p_2 is the split-step result at the $z + \Delta z$ step.

Higher Order FKX Methods

Clearing fractions in

$$k_z^{fd} = \frac{2(k_x^2 + k_y^2)}{4\frac{\omega^2}{v_0^2} - 3(k_x^2 + k_y^2)} \omega \Delta s^2$$

and then gathering up all the terms into **A**, and **B** factors produces

$$\mathbf{A} p(k_x, k_y, z + \Delta z, \omega) = \mathbf{B} p_2(k_x, k_y, z + \Delta z, \omega)$$

so that solving for p we get the implicit propagation formula

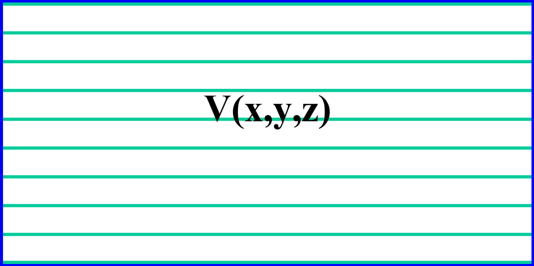
$$p(k_x, k_y, z + \Delta z, \omega) = \mathbf{A}^{-1} \mathbf{B} p_2(k_x, k_y, z + \Delta z, \omega)$$

As was the case for one-way finite difference schemes, computing $\mathbf{A}^{-1}$ is not trivial. Considerable research has provided an efficient and accurate approach. Consequently, this methodology is the current state-of-the art for one-way schemes.

Higher Order FKX Methods

- Slice-by-slice
  - Step 1 in FK
  - Step 2 in FX
  - Step 3 in FK
    - Implicit

Note: Inversion of the A factor is non-trivial



Higher Order FKX Methods.

Boundaries for Frequency Domain Methods

Transform length is sufficiently padded in order to assure waves pass beyond reflection events. Basically the model is projected linearly beyond the actual model boundary.

Outline

1. **Two-Way Equations**
   - The Pseudo Spectral Method
2. **One-Way Wave Equations**
   - Phase Shift and Phase Screen Methods
   - Boundaries
3. **Finite Elements**
   - Two Way Finite Element Approximations
   - Boundary Layers
4. **Raytrace Methods**
   - Snell
   - The Eikonal and Transport Equations
   - Dynamic Raytracing
   - Gaussian Beams

Variational Formulation in Frequency

We begin with the 3D variable density pressure wave equation in the frequency domain.

$$\frac{k^2}{\rho} + \frac{1}{\rho}\left[\frac{\partial}{\partial x}\frac{1}{\rho}\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\rho}\frac{\partial u}{\partial y} + \frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial u}{\partial z}\right] = s(x_0, y_0, z_0, \omega)$$

where $k^2 = \frac{\omega^2}{v^2}$ and $s(x_0, y_0, z_0, \omega)$ is a pressure source located at (x_0, y_0, z_0).

Variational Formulation in Frequency

The previous equation can be put into what is called the variational form

$$\phi(u,w) = \int\limits_{\Omega} \frac{k^2}{\rho} uw d\Omega + \int\limits_{\Omega} \frac{1}{\rho} \nabla u \cdot \nabla w d\Omega = -s$$

where w is an element of a suitable space W of functions that can be used to approximate $u(x,y,z)$. In many cases, the $w(x,y,z)$ functions are polynomials of some order, but can be very general. In this case we think of Ω as the velocity grid or Earth model.

Finite Element Basis Functions

Given a family, w_k, of functions from W, we approximate u, and s by $u(x,y,z,\omega)=\sum_{k=1}^{n} A_k w_k(x,y,z,\omega)$ and $s(x_0,y_0,z_0,\omega)=\sum_{k=1}^{n} b_k W_k(x_0,y_0,z_0,\omega)$ so that the previous formulation becomes

$$\sum_{k=1}^{n} A_k \phi(w_k, w_j) = \sum_{k=1}^{n} b_k \int_{\Omega} w_k w_j d\Omega$$

In this formulation, we are trying to determine the A_k coefficients. Once we have them we will be able to compute $u(x,y,z,\omega)$ by the above formula. Since we know $S(x_0,y_0,z_0,\omega)$ we know the b_k coefficients. Thus for each ω the formula above takes the discrete form

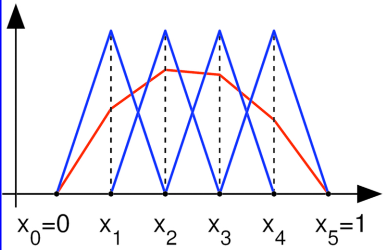
$$\mathbf{S}\tilde{\mathbf{A}} = \mathbf{M}\tilde{\mathbf{b}}$$

where $\mathbf{S} = \left[\phi(w_k, w_j)\right]$, $\tilde{\mathbf{A}} = \left[A_k\right]^T$, and $\mathbf{M} = \left[\int_{\Omega} w_j w_k d\Omega\right]$ and $\tilde{\mathbf{b}} = \left[b_k\right]^T$

Finite Element Basis Functions

As an example, consider the one dimensional case and let

$$w_k(x) = \begin{cases} \frac{x - x_{k-1}}{x_k - x_{k-1}} & x \in [x_{k-1}, x_k] \\ \frac{x_{k+1} - x_k}{x_{k+1} - x_k} & x \in [x_k, x_{k+1}] \\ 0 & \textit{elsewhere} \end{cases}$$



The function w_k is the unique function of position whose values is 1 at x_k and zero at every x_j with $j \neq k$. Basis of this type are easily extended to higher dimensions. As illustrated in the figure the $w_k = 1, ..., n$ basis in (blue) when appropriately scaled and summed produces an approximation (red line) of some desirable function.

Finite Element Basis Functions

The primary advantage of this choice of basis is that the inner product

$$\langle w_j, w_k \rangle = \int_0^1 w_j w_k \, dx$$

will be zero for almost all j,k. As long as x_j and x_k do not share an edge of the triangulation, then

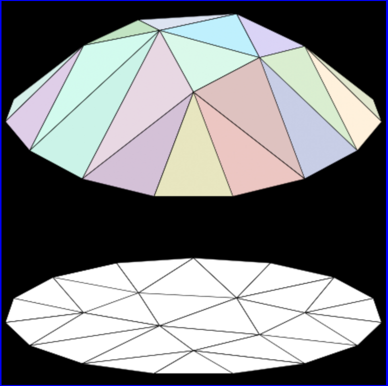
$$\int_\Omega w_j w_k \, ds = 0$$

and

$$\int_\Omega \nabla w_j \cdot \nabla w_k \, ds = 0.$$

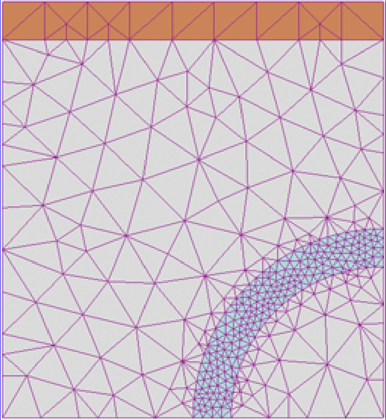
This finite element scheme simplifies the **M** matrix an makes its inversion feasible.

Finite Element Approximations



The basis functions w_k need not be simple, but for seismic finite elements methods they almost always are. In this figure the domain Ω has be split up into triangles. Its quite easy to visualize replacing the w_k functions with pyramids as elements to approximate a dome like surface. In this case the triangular mesh used for the Earth model Ω is regular. (From Wikipidia)

Finite Element Approximations



This figure shows an Ω dome (a 2D Earth Model) expanded into a non-regular triangular mesh. While pyramids could still be used as the w_k functions, the Earth model in this form would be much more complex then one with an absolutely regular grid. The complexity of Finite element meshes in the geophysical case is determined by the complexity of the Earth model.

Discussion

The key point to this discussion is that we have wide latitude in the choice of the w_k. We can divide the model into a collection of local regions and then define the w_k through polynomials, pyramids, or perhaps even boxes, over each of the sub domains. This approach tends to work well when the problem is defined by thing like bridges, aeronautical structures, and other more rigid bodies, but has never gained ground in seismic settings. Mathematically, dividing the medium up in this way is quite difficult so more modern approaches tend to choose domains that are uniformly square or rectangular. Its also convenient to have basis functions that are easily differentiable and orthogonal Thus, in some sense the modern version of FEM tends to look more and more like a very sophisticated finite difference approach.

Finite Element Matrix Equation

Regardless of how one chooses the basis functions, the discrete version of the targeted wave equation can always be expressed in matrix form

$$\mathbf{S}\vec{A} = \mathbf{M}\vec{b}$$

where $\vec{A}^T = [A_1, A_2, \cdots, A_n]$ $\vec{b}^T = [b_1, b_2, \cdots, b_n]$.
In this case, **S** is the complex impedance matrix, and **M** the stiffness matrix. Note that reference to frequency ω has been dropped so that this equation must be solved frequency-by-frequency.

Finite Element Discretization

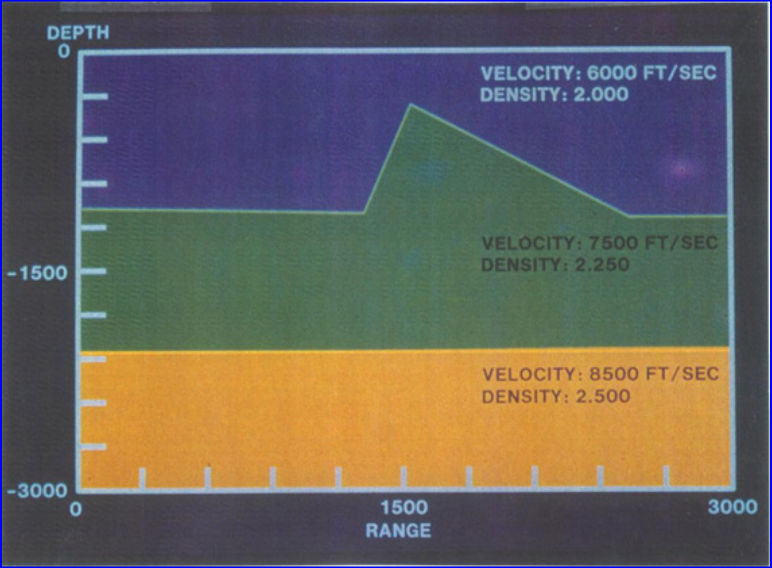
If we choose our mesh scheme properly we may always assume that $\mathbf{S}$ is square, symmetric, and invertible. so that the modeling operator $\mathbf{S}^{-1}$ generates data according to the formula

$$\vec{U} = \mathbf{S}^{-1}\mathbf{M}\vec{b}$$

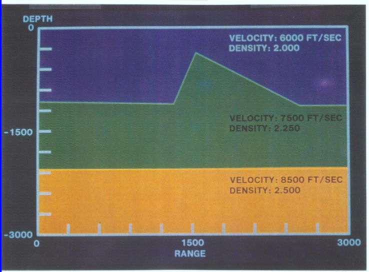
Matrix and approximation Issues

- Frequency by frequency approach
  - Can easily limit the number of frequencies one inverts
  - Each frequency inversion provides an image
- All sources can be generated from one matrix inversion
- Matrix is huge in 3D
  - Extremely difficult to invert
  - Recent progress suggest this may become possible in the future
- High quality approximation yields high quality results
  - Can be much more accurate than other methods

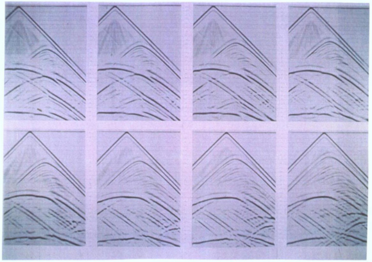
An Old Finite Element Movie



Pyramid Model and Data



(a) A simple acoustic velocity model



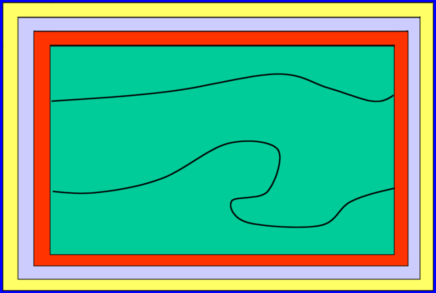
(b) Eight shots using the model in (a)

A simple pyramid model and data. The finite element data over this model was synthesized on VAX 11-780 computes in late 1981 and early 1982. At that time the calculations necessary to compute each shot was on the order of 48 hours. Today most laptops can compute the entire set of 24 shots in seconds.

FDM vs FEM

- The FDM is an approximation to the differential equation
- The FEM is an approximation to its solution.
- The FEM can effectively handle extremely complicated geometries (and boundaries)
- In its basic form the FDM is restricted to rectangular shapes.
  - Vertex effects can be pronounced.
- The FEM requires inversion of a potentially large matrix
- The FDM is simple to implement and explicit. Matrix inversion is normally not necessary.
- The FEM is generally considered to be more mathematically accurate then the FDM
- The quality of a FEM approximation is often higher than in the corresponding FDM approach
  - Extremely problem dependent .
  - Contrary examples abound

Boundaries



Realistic seismic simulations generally include procedures for suppressing boundary reflections. Modern approaches begin by surrounding the model with a small number of fake layers. Modified equations for absorbing energy are then applied layer by layer to produce a desired level of suppression. The number of layers is certainly a function of method but typically ranges from a handful to perhaps ten to fifteen.

Sponge Boundaries for Finite Elements

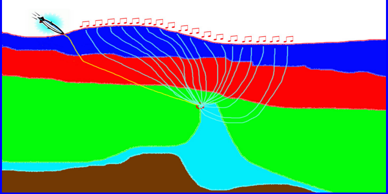
For the finite element method, sponge boundaries can be implement by changing the definition of $\phi(U, V)$ to

$$\phi(U, V) = (1+\alpha) \int_{\Omega} \frac{k^2}{\rho} UV d\Omega + (\beta + 1) \int_{\Omega} \frac{1}{\rho} \nabla U \cdot \nabla V d\Omega$$

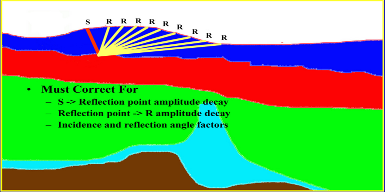
where α and β are the damping factors in each of the boundary layers.

Outline

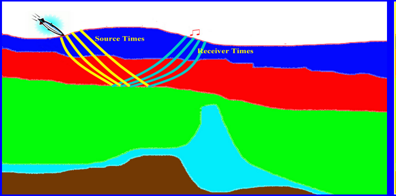
Huygens Principle and Raytrace Modeling



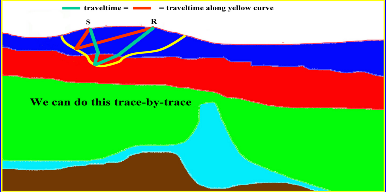
(a) Exploding a point reflector



(b) Raytrace amplitude corrections



(c) Fixed offset raytrace modeling



(d) Equal traveltime modeling

Huygens Principle as it might be applied to raytrace modeling

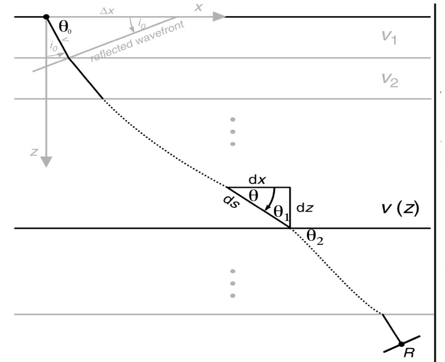
Issues

- Raytracing methods
- How many arrivals does one compute
- How does one compute multiple arrivals
- Top-down vs Bottom-up raytracing
- Amplitude corrections

3 Raytracing Methods

- Pure Snell's law
  - Compute the incidence and transmission angles
- Solving the Eikonal equation along rays
  - Dynamic Raytracing
  - Gaussian Beams
- Solving the Eikonal equation via Finite Difference
  - Normally first arrivals only

Raytracing — Snell



$$\text{Snell's Law} \quad \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

The surface take-off vector is

$$\mathbf{p} = (cos\theta_0, sin\theta_0).$$

Along the ray

$$\frac{\mathrm{d}(x(s), z(s))}{\mathrm{d}s} = (cos\theta, sin\theta)$$

and

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}s} = \frac{1}{2}\left(\frac{\partial v^{-1}(x,z)}{\partial x}, \frac{\partial v^{-1}(x,z)}{\partial z}\right)$$

Raytracing in a $v(z)$ medium.

Wave Equation Rays

For a source at x_s and receiver at x_r if we denote the traveltime or *phase* from x_s to x_r by $\tau(x_r, x_s)$ and the amplitude decay by $A(x_r, x_s)$ we can then write

$$G(x_r, x_s, t) \approx A(x_r, x_s)\delta(t - \tau(x_r, x_s))$$

in space-time and

$$G(x_r, x_s, \omega) \approx A(x_r, x_s)e^{i\omega\tau(x_r, x_s)}$$

in frequency. Substituting into the wave equation

$$\left(\nabla \cdot \nabla - \frac{i\omega^2}{v^2}\right) A(x_r, x_s)e^{i\omega\tau(x_r, x_s)} = 0$$

The Eikonal and Transport Equations

We get

$$\left(i\omega^2\left[(\nabla\tau)^2-\frac{1}{v^2(\mathbf{x})}\right]+i\omega\left[2\nabla A\cdot\nabla\tau+A\Delta\tau\right]\Delta A\right)e^{i\omega\tau}=0.$$

Equating coefficients of powers of $i\omega$ to zero yields the Eikonal equation

$$(\nabla\tau)^2=\frac{1}{v^2(\mathbf{x})}$$

and the transport equation

$$2\nabla A\cdot\nabla\tau+A\Delta\tau=0$$

Note: Ray is a solution to the wave equation, but only along an infinitely thin path.

The Method of Characteristics

The method of characteristics finds Eikonal traveltimes by solving

$$\frac{d\mathbf{x}}{ds} = \mathbf{p}$$

and

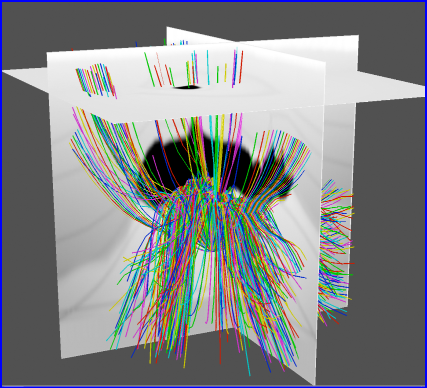
$$\frac{d\mathbf{p}}{ds} = \nabla\left(\frac{1}{2v(\mathbf{x}(\mathbf{s}))}\right)$$

where the initial take-off vector is

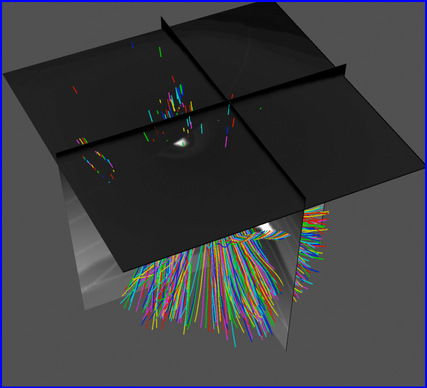
$$\mathbf{p}(0) = \frac{1}{v(\mathbf{x}(0))}\begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix},$$

s is arc length,$\mathbf{x}(s)$ is the position vector, $\mathbf{x}(0)$ is the initial source position, θ is the azimuth, and ϕ the declination of the ray

Rays Along Characteristic Curves



(e) Upward traveling rays



(f) Emerging upward traveling rays

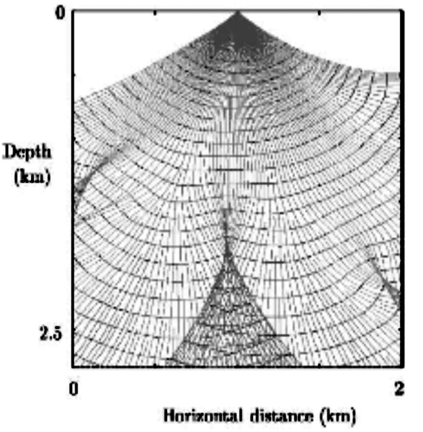
Upward raytracing in complex salt

Finite Difference Eikonal

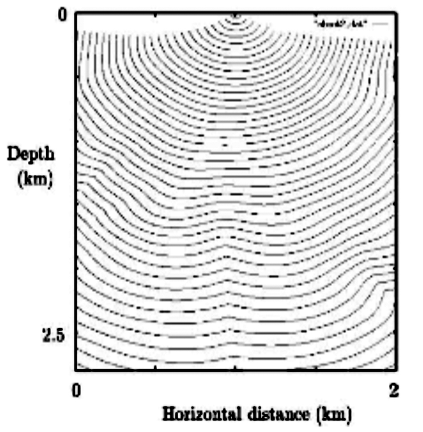
$$\frac{\partial \tau}{\partial z} = \sqrt{\frac{1}{v^2(\mathbf{x})} - \frac{\partial \tau}{\partial x} - \frac{\partial \tau}{\partial y}}$$

Is easily solved by the same tricks used to solve one-way equations.: Approximate the square root and then derive a finite difference formula for τ. While the method is extremely efficient it has some of the same issues that one-way solutions have. Its not possible to generate turning rays and its very difficult to include anisotropy.

Ray vs Eikonal Fans



(g) Raytrace traveltime fan

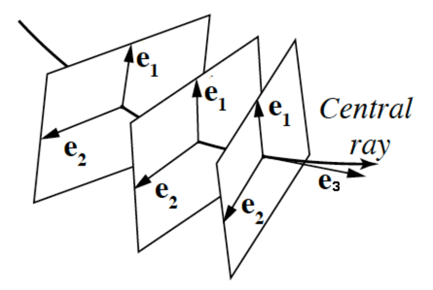


(h) Eikonal traveltime fan

Raytrace vs Eikonal traveltime fans. After Hermann et. al. 2000

Dynamic Rays

- Given a *central* ray
    - Is it possible to efficiently propagate amplitudes along the ray



Dynamic Rays

Assume that off the ray where $q_1 = q_2 = 0$ the solution is

$$x(q_1, q_2, s) = x(0, 0, s) + q_1\mathbf{e_1} + q_2\mathbf{e_2}$$

Following the procedure used to get the first Eikonal equation, substitute

$$u(q_1, q_2, s, \omega) \approx A(q_1, q_2, s)e^{i\omega\tau(q_1, q_2, s)}$$

into the wave equation. The result is another Eikonal of the form

$$\frac{\partial\tau}{\partial s} = \frac{1 + \frac{v_{0,1}}{v_0}q_1 + \frac{v_{0,2}}{v_0}q_2}{v}\sqrt{1 - v^2\left[\frac{\partial\tau}{\partial q_1}^2 + \frac{\partial\tau}{\partial q_2}^2\right]}$$

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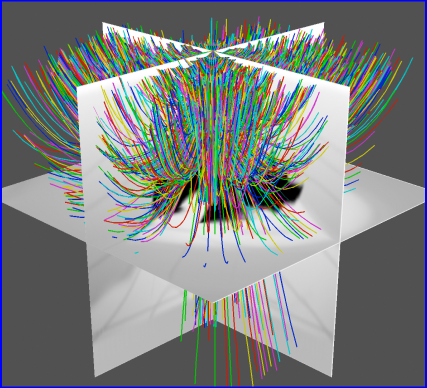
Dynamic Rays

The mathematics at this point becomes extremely messy and beyond the scope of these notes but basically with a few more tricks, one can propagate an initial amplitude along the ray. The result is a ray that satisfies the wave equation kinematically as well as dynamically.

- Derive two auxiliary equations
  - Something like the transport equations
- Simultaneously solve the Eikonal and two auxiliary equations for $\tau(q_1, q_2, s)$
- Use the auxiliary equations to compute $A(0, 0, s)$ on the ray

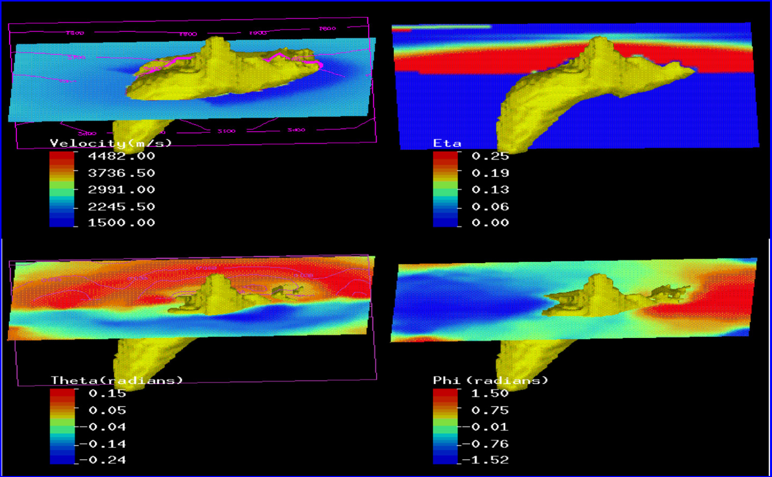
Dynamic Ray Tracing

- Derived from WEQ
- Multiple arrivals/paths
- Information on the ray
  - Traveltime and Amplitudes
    - Amplitudes only along the ray
    - No way to determine validity of the amplitude off the ray



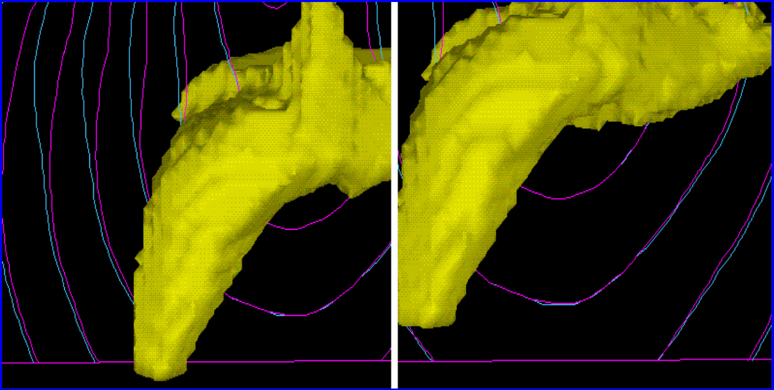
Downward rays in complex salt

Anisotropic Raytracing



Anisotropic model. Clockwise from top right: Vertical Velocity, η, symmetry axis *theta*, symmetry dip angle ϕ.

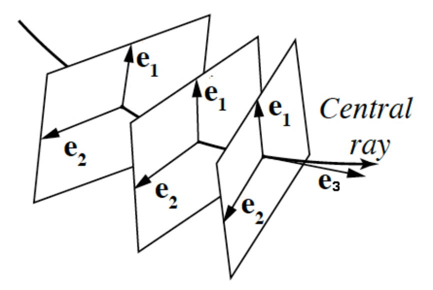
Anisotropic Raytracing



Anisotropic raytraced based wavefronts from the previous model.

Gaussian Beams

- Given a *central* ray
  - Is it possible to construct a solution to the wave equation in the neighborhood of a ray path



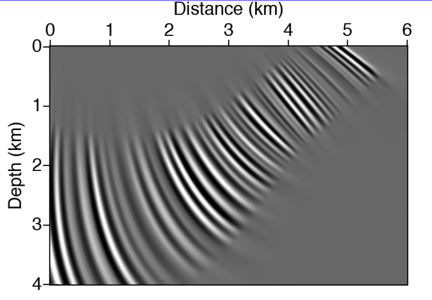
Gaussian Beams

In this case the mathematics becomes even more complex but the process is virtually identical to that used for dynamic raytracing

- Derive two auxiliary equations
  - Something like the transport equations
- Determine appropriate complex initial conditions
- Simultaneously solve the Eikonal and two auxiliary equations for $\tau(q_1, q_2, s)$
  - Use the auxiliary equations and traveltime to compute $A(q_1, q_2, s)$
  - Orthogonally, $A(q_1, q_2, s)$ decays as a Gaussian
  - The imaginary part of the traveltime determines the weight at each s

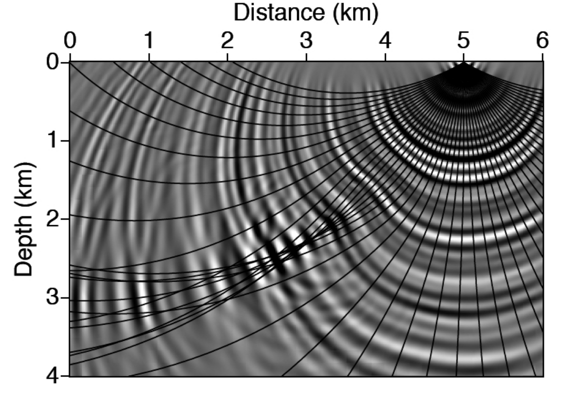
Gaussian Beams

- Derived from WEQ
- Multiple arrivals/paths
- Information on and off the ray
  - Traveltime and Amplitudes
    - Amplitudes decay orthogonally



A single Gaussian Beam or Fat Ray. Amplitudes off the central ray are determined by a frequency domain Gaussian Bell. After Hale 1993

Gaussian Beams



Gaussian Beam Forward Modeled Shot. Multiple Gaussian Beams are summed together to produce a one-way forward propagated shot. After Hale 1993

Questions?

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