Seismic Modeling, Migration, and Velocity Inversion

The Rocks, Their Parametrization, and Modeling

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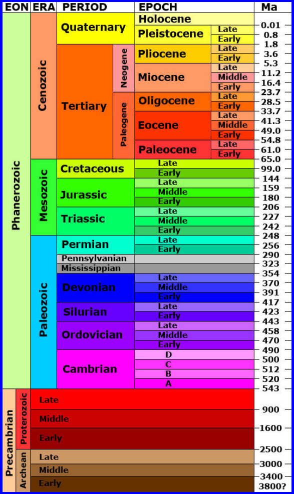
Outline

1. **Earth Model Parameters**
   - Geology
   - Mathematical Physics
   - Isotropic
   - Isotropic Elastic
   - Anisotropic
     - Vertical Transverse Isotropy
     - Tilted Transverse Isotropy
     - Orthorhombic Isotropy
   - Anisotropic Indicators
2. **Synthetic Earth Models**
   - Stratigraphic and Structural Styles
   - Predicting Stratigraphy from Logs
     - Well Log Statistics
     - Well log Prediction
     - Adding Reservoirs
   - Adding Structure by Inverse Flattening
   - Reflectivity from Imaged data

Outline

Broad Descriptive View

- Historical
  - Miocene, Oligocene, Eocene, Paleocene, Cretaceous, · · · , Silurian, · · ·
  - The really really old Precambrian · · ·
    - Proterozoic, Archean, Formation of the Earth
- Mechanisms
  - Clastics, Carbonates, Evaporites,Metamorphosed, · · ·
- Structural
  - Plate tectonics, Erosion, Heating, Cooling · · ·



The wave equation

$$\frac{1}{v^2(x,y,z)}\frac{\partial^2 U}{\partial t^2} - \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) = S(x_0, y_0, z_0, t)$$

- How does one parametrize rocks?
  - By Deriving a mathematical model describing sound propagation in rocks
    - The mathematical model incorporates the proper parameters
    - It is called the wave equation
- The wave equation is used to:
  - Synthesize realistic synthetic seismic data (run forward)
    - By Raytracing, finite difference, finite elements $\cdots$
  - Migrate or image seismic data (run backward)
    - Kirchhoff, Beam, WEM, and RTM

Isotropic Earth

In an isotropic Earth

- Derivation of the mathematical model results in two parameters
  - ρ and v_p
- v_p does not vary as a function of angle
- There is one compressional wavefield
  - The compressional wavefield propagates orthogonal to the rock fabric

An Acoustic Earth thus consists of just two volumes, velocity, v_p, and density, ρ. In this case, the wavefield has just one *scalar* component.

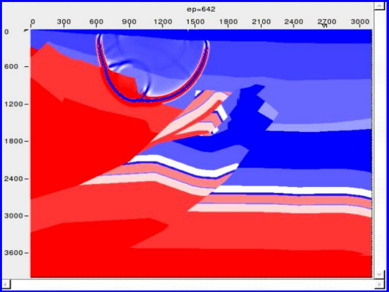
Isotropic

The isotropic velocity is characterized by a Lamé parameter, λ

The velocity is

$$v_p = \sqrt{\frac{\lambda}{\rho}} = \sqrt{\frac{K}{\rho}}$$

Where K is the bulk modulus



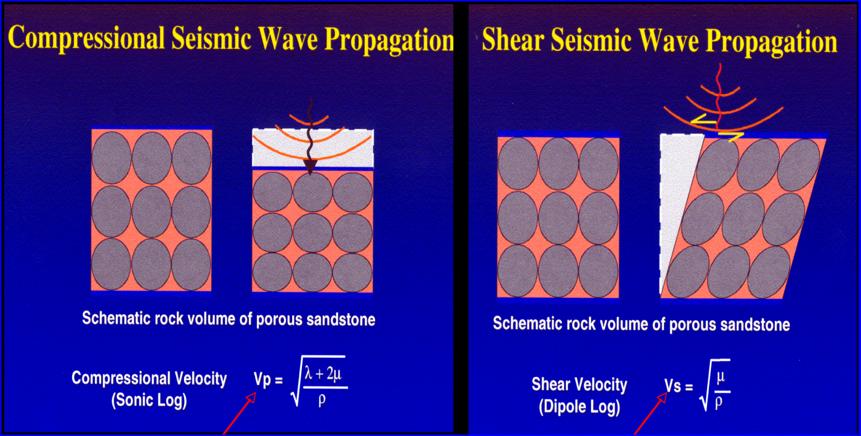
Play Movie

Isotropic Elastic

In an isotropic elastic Earth,

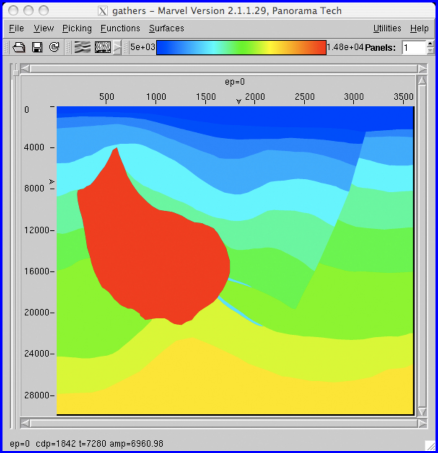
- Derivation of the mathematical model results in three parameters
  - ρ and v_p and v_s
- There is one compressional and one orthogonal shear wavefield
- Neither v_p nor v_s varies as a function of angle
  - The compressional wavefield propagates normal to the rock fabric
  - The shear wavefield propagates parallel to the rock fabric
  - The two wavefields are always orthogonal to each other
  - The two waves constantly convert from one to the other

Compressional vs Shear Propagation

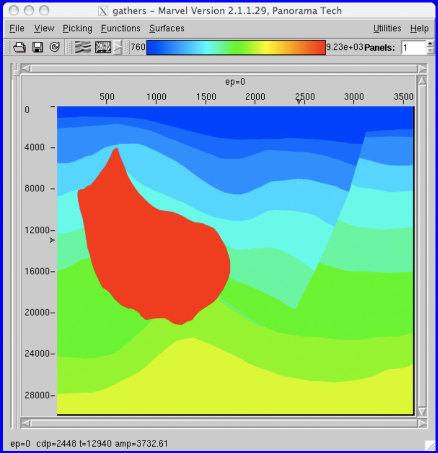


Compressional propagation takes places normally to the rock fabric . In contrast, shear wave propagation takes place horizontally. As indicated here the compressional and shear velocities are determined by density, ρ, and two Lamé parameters λ and μ.

Isotropic Elastic Movie

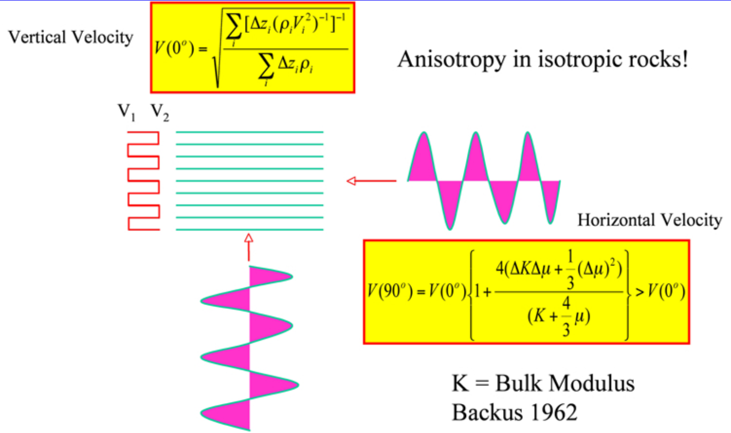


(a) v_p



(b) v_s

Vertical vs Horizontal Velocities in Thin Layers



In 1962 M. Backus showed mathematically that when rocks are thinly layered the horizontal velocity is not equal to the vertical velocity. Thus thinly layered rocks are always anisotropic.

Anisotropic Rocks

In an anisotropic Earth,

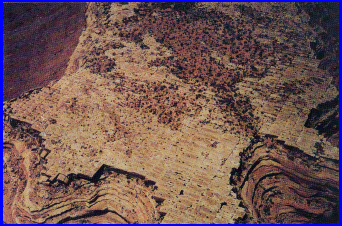
- The derived mathematical model has 21 independent parameters
- There is one compressional and two shear wavefields
- Each velocity varies as a function of angle
  - The compressional wavefield propagates normal to the rock fabric
  - Each of the two shear waves propagate parallel to the rock fabric
  - The shear waves propagate orthogonally to each other at all times
  - The central axis is determined by the compressional wave
  - The three waves constantly convert from one to the other

The REALLY BIG question: How does one parametrize an UNDERSTANDABLE version of such a model?

Anisotropic or Fully Elastic Earth

The 21 independent volumes are usually arranged in the symmetric $C = \left[c_{ij}(x,y,z)\right]$ matrix:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix}$$



Notes:

- For every ij pair, $\sqrt{\frac{c_{ij}}{\rho}}$ is a velocity
- Each such velocity governs propagation in a preferred direction
  - Propagation is very complex and difficult to understand
- Anisotropy in the Earth is usually weak ($\approx$ -5% $\rightarrow$ 10%)

Simplifications

Some simplifying assumptions:

- Velocities of propagation have vertical and horizontal components
  - The vertical velocity represents a vertical log
  - The horizontal velocity represents a horizontal log
  - Both velocities are true rock velocities.
- The propagation exhibits some form of symmetry
  - Velocities are tied together via an angle
- The anisotropy is weak ($\approx$ -10 to 15 percent)

Vertical Transverse (Polar) Isotropy (Weak)

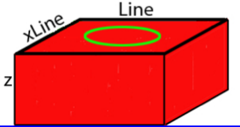
An understandable spherically symmetric anisotropic model can be defined in terms of a single angle θ as follows:

$$v_p(\theta) \approx v_{p_{vert}} \left[1 + \delta \sin^2\theta \cos^2\theta + \epsilon \sin^4\theta\right]$$

$$= v_{p_{vert}} \left[1 + \delta \sin^2\theta + (\epsilon - \delta) \sin^4\theta\right]$$

$$v_{s_\perp}(\theta) \approx v_{s_{\perp vert}} \left[1 + \frac{v^2_{p_{vert}}}{v^2_{s_{\perp vert}}} (\epsilon - \delta) \sin^2\theta \cos^2\theta\right]$$

$$v_{s_\parallel}(\theta) \approx v_{s_{\perp vert}} \sqrt{1 + 2\gamma \sin^2\theta}$$



This is a vertically transverse isotropic or polar anisotropic (Thomsen, Geophysics 1984) model where

- δ is the near vertical anisotropy percentage
- ϵ is the near horizontal anisotropy percentage
- γ is the near vertical shear anisotropy percentage

The c_{ij} parameters and Thomsen notation

Thomsen as c_{ij}

$$v_{p_{vert}} = \sqrt{\frac{c_{33}}{\rho}}$$

$$v_{s_{\|vert}} = \sqrt{\frac{c_{44}}{\rho}}$$

$$\epsilon = \frac{c_{11}-c_{33}}{2c_{33}}$$

$$\delta = \frac{(c_{13}+c_{44})^2-(c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})}$$

$$\gamma = \frac{c_{66}-c_{44}}{2c_{44}}$$

The C matrix

$$\begin{bmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11}-2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

In this case the mathematical model for Polar or VTI anisotropy is parametrized by six independent volumes.

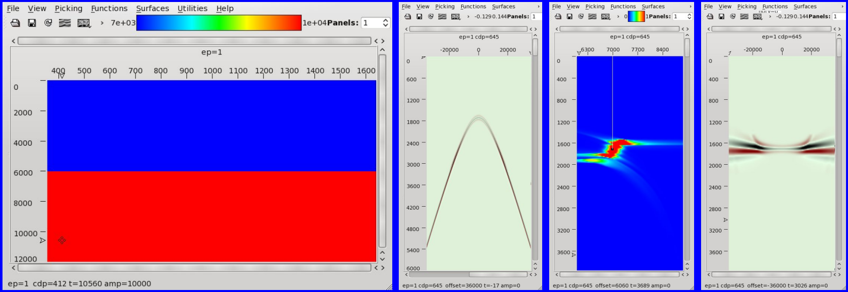
Anisotropy in terms of Velocities

With $v_{p_{horz}} = v_p(90)$ and $v_{s_{\|\,horz}} = v_{s_{\|}}(90)$ we can express ϵ and γ in terms of horizontal and vertical velocities:

$$\begin{aligned} \epsilon &\approx \frac{v_{p_{horz}}}{v_{p_{vert}}} - 1.0 \\ \gamma &\approx 0.5 * \left(\frac{v_{s_{\|\,horz}}}{v_{s_{\perp\,vert}}}\right)^2 - 0.5 \end{aligned}$$

However, a similar equation for δ requires a bit more analysis

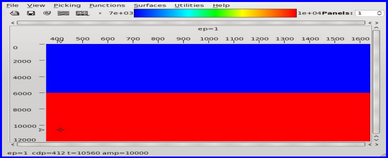
A Simple $v(z)$ Isotropic Model



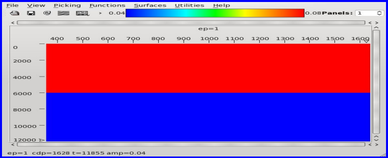
(a) VP (b) CDP (c) SEM (d) NMO

A simple two layer $v(z)$ isotropic model and synthetic CDP. The v_p of the top layer in (a) is 7000 ft/sec, while v_p in the bottom layer is 10,000 ft/sec. The offset range for the cdp in (b) is $\pm$36,000 ft. Velocity in the semblance panel in (c) ranges from 6000 to 9000 ft/sec. Note the relative flat response in the NMO'd gather shown in (d). The up-sweep is due to NMO stretch.

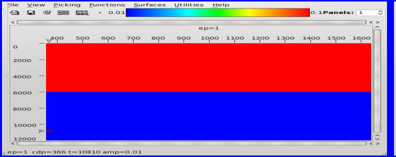
A simple $v(z)$ VTI model



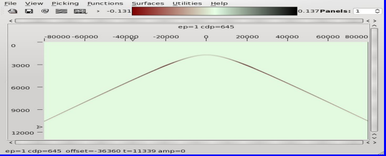
(a) VP



(b) δ



(c) ϵ



(d) CDP

A simple two layer $v(z)$ VTI model and synthetic CDP. The v_p velocity in (a) is the same as in the previous slide. The value of δ in the top layer of (b) is 8% while the value of ϵ in the top layer of (c) is 10%. The offset range of the split-spread in (d) is $\pm$80,000 ft.

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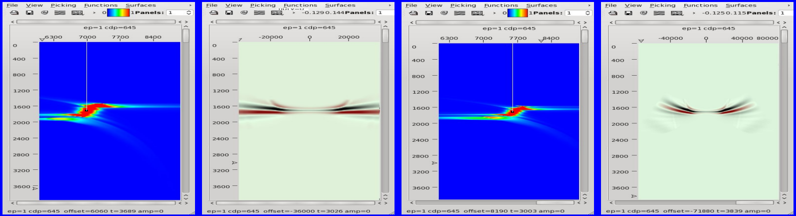
NMO Analysis for Isotropic and VTI Gathers



(a) ISO-SEM (b) ISO-CDP (c) VTI-SEM (d) VTI-CDP

Comparison of NMO analysis for an isotropic and an anisotropic (VTI) gather. In (a) and (b) we see reasonably flat correction at the correct (7000 ft/sec) vertical velocity, while in (d) correction for the near offsets requires a pick of roughly 7,600 ft/sec. This is consistent with $\delta = 8\%$. Note the long offset sweep-up in (d). Thus for a flat layer the *p* moveout velocity is approximately:

$$v_{p_{nmo}} \approx v_{p_{vert}} (1 + \delta)$$

Vertical vs Horizontal Velocities

In general, $v_{p_{nmo}}$ is defined to be that velocity which produces the optimum isotropic migration. Its what we believe we get when we estimate isotropic interval velocities in depth imaging projects. Given $v_{p_{nmo}}$ Thomsen parameters for Polar or VTI anisotropic models are given by:

$$\epsilon \approx \frac{v_{p_{horz}}}{v_{p_{vert}}} - 1.0$$

$$\gamma \approx 0.5 * \left(\frac{v_{s_{\|horz}}}{v_{s_{\perp vert}}}\right)^2 - 0.5$$

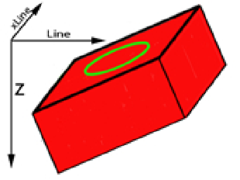
$$\delta \approx 0.5 * \left(\frac{v_{p_{nmo}}}{v_{p_{vert}}}\right)^2 - 0.5$$

Rocks Exhibiting VTI Anisotropy

- Thin-bed sequences
- Thinly laminated shale
- Rocks with a single set of vertical fractures
- Rocks with a single set of non circular fractures

Tilted Transverse Isotropy (TTI)

Tilted Transverse Isotropy defines rocks whose symmetry axis at any given point in the Earth is not vertical. Thus our compressional and shear velocities are functions of two angles $\theta(line, cdp, z)$ and $\phi(line, cdp, z)$ defining the azimuth and the dip at each point in the subsurface. The equations that specify this kind of anisotropy are similar to those of previous slides.



TTI Anisotropy

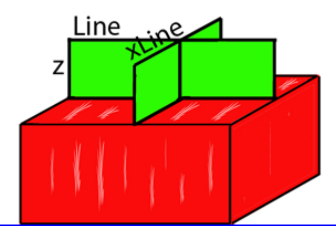
Examples of rock types that exhibit TTI symmetry are essentially similar to those exhibiting VTI symmetry. Thus, they include

- Thin-bed sequences or shale with a single set of vertical fractures
- Isotropic formations with a single set of vertical, non circular fractures

Typically, the symmetry angles are proportional to bed dip. In the Gulf of Mexico, it has been found that TTI symmetry is proportional to local dip. Unfortunately, determining the proportionality factor usually requires iterating the depth migration process.

Orthorhombic Media

- $V_{p_{vert}}$ – vertical P-wave velocity
- $V_{s||_{vert}}$ – vertical S-wave velocity
  - Line Polarization
- $V_{s\perp_{vert}}$ – vertical S-wave velocity
  - xLine Polarization
- $\epsilon^{(1)}$ – VTI ϵ in xline-depth plane
- $\epsilon^{(2)}$ – VTI ϵ in line-depth plane
- $\delta^{(1)}$ – VTI δ in xline-depth plane
- $\delta^{(2)}$ – VTI δ in line-depth plane
- $\delta^{(3)}$ – VTI δ in line-xline plane
- $\gamma^{(1)}$ – VTI γ in xline-depth plane
- $\gamma^{(2)}$ – VTI γ in line-depth plane



Orthorhombic models are those characterized by three mutually orthogonal symmetry planes. For such media the Thomsen parameters must be extended to include each such plane.

Thomsen In Terms of the C Matrix

- $V_{p_{vert}} = \sqrt{\frac{c_{33}}{\rho}}$
- $V_{s\|_{vert}} = \sqrt{\frac{c_{55}}{\rho}}$
- $V_{s\perp_{vert}} = \sqrt{\frac{c_{44}}{\rho}}$
- $\epsilon^{(1)} = \frac{c_{22}-c_{33}}{2c_{33}}$
- $\epsilon^{(2)} = \frac{c_{11}-c_{33}}{2c_{33}}$
- $\delta^{(1)} = \frac{(c_{23}+c_{44})^2-(c_{33}-c_{44})^2}{2c_{33}(c_{33}-c_{44})}$
- $\delta^{(2)} = \frac{(c_{13}+c_{55})^2-(c_{33}-c_{55})^2}{2c_{33}(c_{33}-c_{55})}$
- $\delta^{(3)} = \frac{(c_{12}+c_{66})^2-(c_{11}-c_{66})^2}{2c_{11}(c_{11}-c_{66})}$
- $\gamma^{(1)} = \frac{c_{66}-c_{55}}{2c_{55}}$
- $\gamma^{(2)} = \frac{c_{66}-c_{44}}{2c_{44}}$

Orthorhombic models can be expressed in terms of velocities but each symmetry plane requires its own set.

Orthorhombic Anisotropy's C Matrix

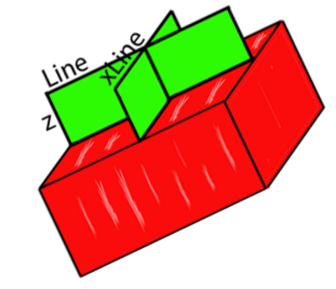
Orthorhombic anisotropy is defined through the 9 element $C = [c_{kl}]$ matrix

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

Orthorhombic rocks are probably the most prevalent, but as far as the author knows there has been little actual use of this kind of anisotropy in real exploration efforts. However, orthorhombic imaging is being offered as a service.

Tilted Orthorhombic Isotropy (TORT)

Tilted orthorhombic isotropy defines rocks whose symmetry axis at any given point in the Earth is not vertical. Thus our compressional and shear velocities are functions of two angles $\theta(line, cdp, z)$ and $\phi(line, cdp, z)$ defining the azimuth and the dip at each point in the subsurface. The equations that specify this kind of anisotropy are similar to those of previous slides. Once again it is thought that the local rock dip determines these angles.

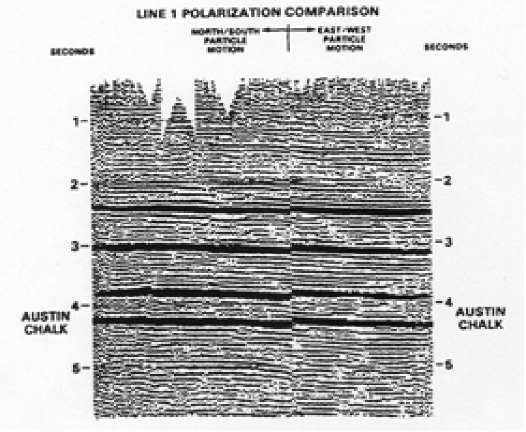


Orthorhombic Anisotropy

Examples of rock types that exhibit orthorhombic symmetry include

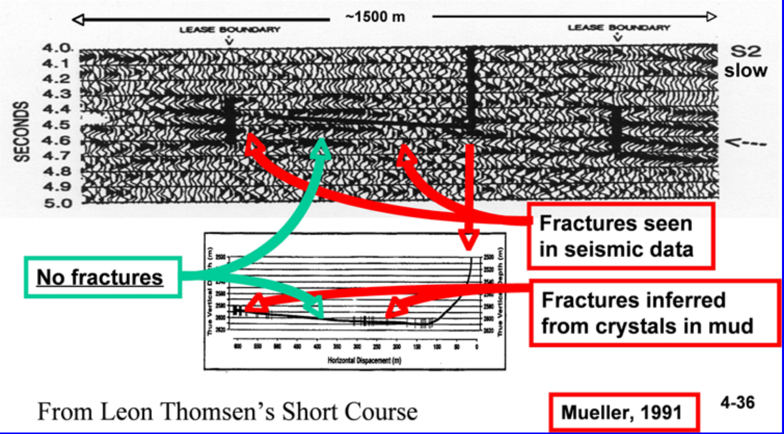
- Thin-bed sequences or shale with a single set of vertical fractures
- Thin-bed sequence, or shale, or a massive isotropic sandstone with orthogonal sets of vertical fractures
- Isotropic formation with a single set of vertical, non circular fractures

Anisotropic Indicators (After Mueller, 1991)



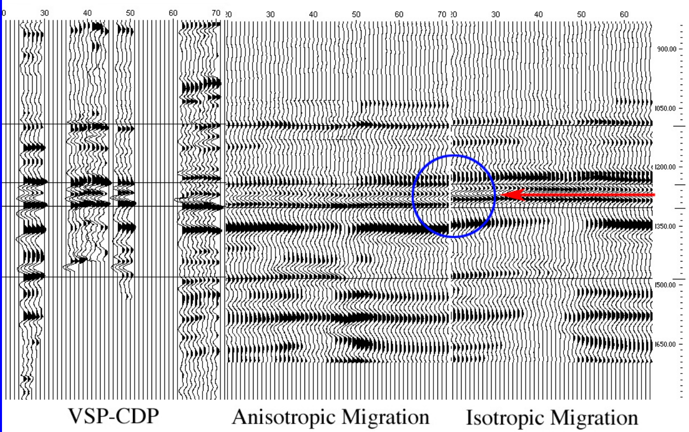
Two-dimensional-time-domain shear sections over the Giddings, Texas Field. The time differences indicate anisotropy because they confirm that one of the two shear sound velocities is faster then the other.

Anisotropic Indicators (After Mueller, 1991)



Shear wave fracture indicators at the Giddings, Texas Field. In this case, the lack of reflectivity (amplitudes) indicates the presence of fractures.

Anisotropic Indicators



Processed VSP (Alberta Basin) with anisotropic and isotropic time migrations. Once again time differences between two different migration algorithms demonstrate the presence of anisotropy.

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   - Anisotropic Indicators

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   - Stratigraphic and Structural Styles
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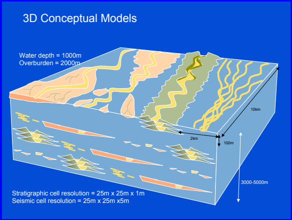
     - Adding Reservoirs
   - Adding Structure by Inverse Flattening
   - Reflectivity from Imaged data

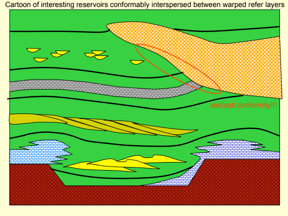
Earth Model Composition

Real World Styles are a composition of

- Stratigraphic
  - Generally relatively flat
    - Highly dipping faults
  - Sedimentary
    - Formational
- Structural Styles
  - Moderate to high dips
  - Folded, mutilated, spindled, and bent
    - Tectonics
    - Salt flow
- Reservoirs
  - From flat to vertical

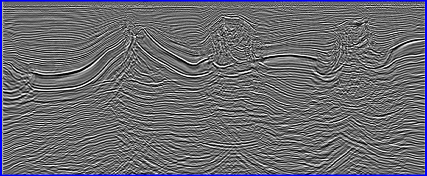


(a) Stratigraphy

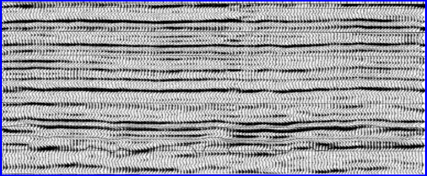


(b) Structure

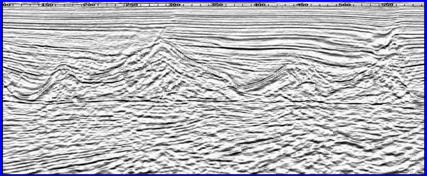
Real World Styles



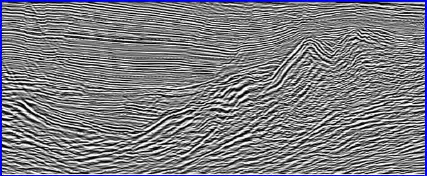
(a) Offshore China - Marine



(c) Alberta Canada -Land



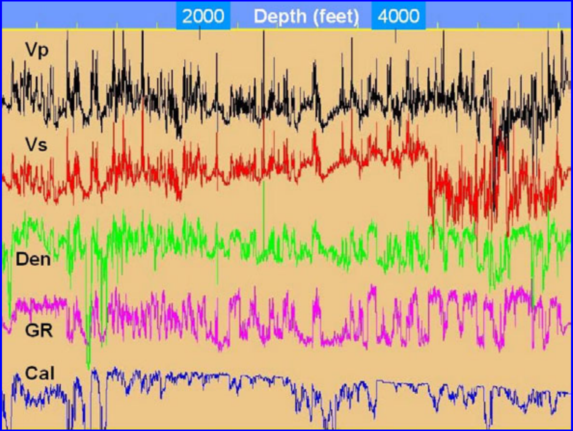
(b) Offshore Africa - Marine



(d) Qianshan China - Land

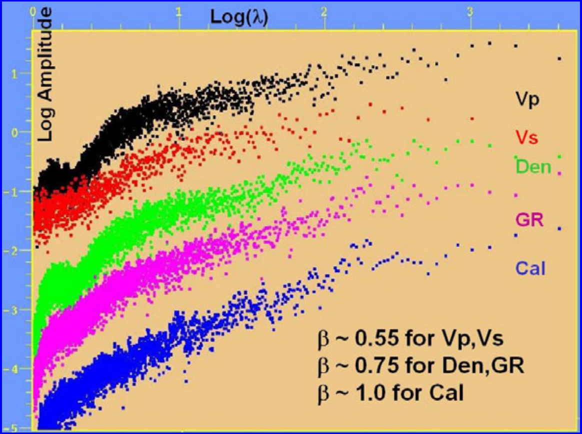
- Each image has both stratigraphic and structural elements
- Structure formed by physical forces
- Useful for construction of synthetic Earth models

Gulf of Mexico Logs — After Joe Stefani 2009



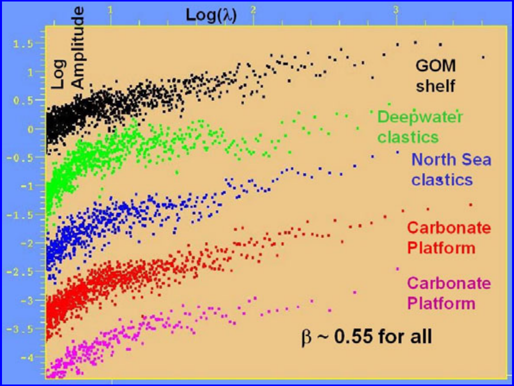
- Drift removed logs
- Note similarity of v_p and v_s logs and the ρ and γ ray logs
- The caliper has some correlation with the latter two

Gulf of Mexico Statistical Log Spectra



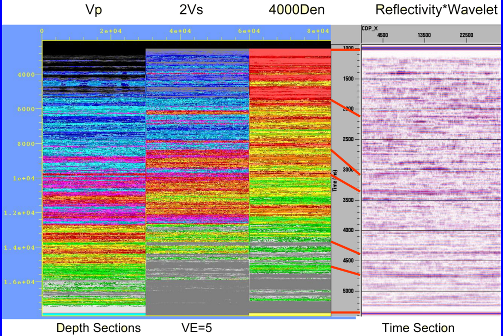
- Fourier domain log-log spectra of several nearby Gulf of Mexico well logs
- β is a measure of the statistical variation of the various spectra
- v_p and v_s and ρ and γ ray show good correlation

Statistical Log Spectra from Various Basins



- Similar v_p log-log well log spectra from various basins
- Statistical similarity is the same for each basin
- Worldwide sedimentation process has the similar statistical character
- Straightforward generation of other nearby realistic synthetic logs

Predicting Well Logs — After Joe Stafani

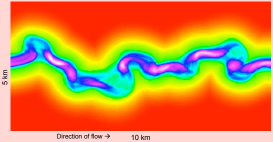


- Statistically generated v_p, $2v_s$, and ρ logs
- Right hand side is a short synthetic normal incidence reflectivity section

Some Interesting Reservoirs

Channels with Levies and Downslope-Migrating Loops

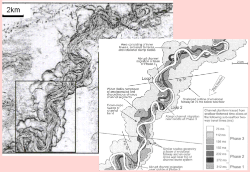
Plan view of a vertical average of Vshale: (white=0, red=1)



Cellular resolution: dx = dy = 25m, dz = 4m

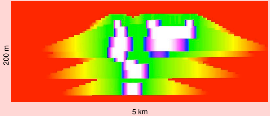
Alternative Slope Valley Analogue

Nigeria, Deptuc et al. 2003



Cross-Section of Channels with Levies Model

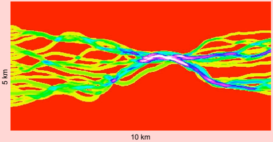
Vshale: (white=0, red=1) Vert Exag = 10:1



Anastomosing & Constricted Channels without Levies

(spaghetti model)

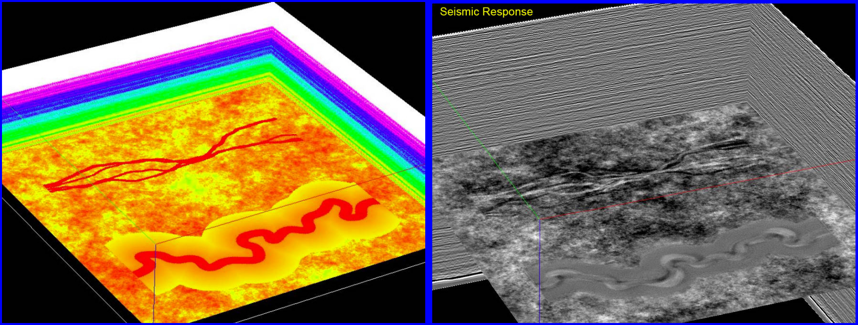
Plan view of a vertical average of Vshale: (white=0, red=1)



Cellular resolution: dx = dy = 25m, dz = 4m

- Two interesting reservoir environments
- Reservoirs constructed using standard Gassman-Biot type equations

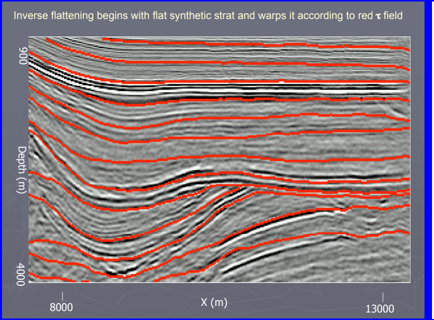
Adding Reservoirs — After Joe Stafani



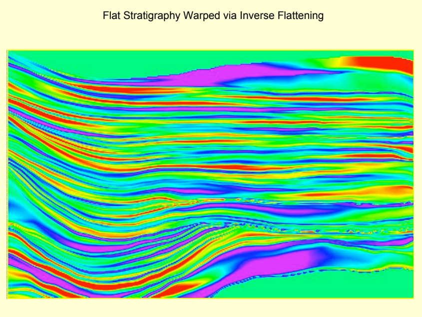
(a) Channel model   (b) Channel model migrated data

- Merge of stratigraphic channel(s) into a synthetic 3D volume
- Background stratigraphy based on statistically predicted logs
- Generation of prestack data for hypothesis testing

Inverse Flattening — After Lomask, et. al.



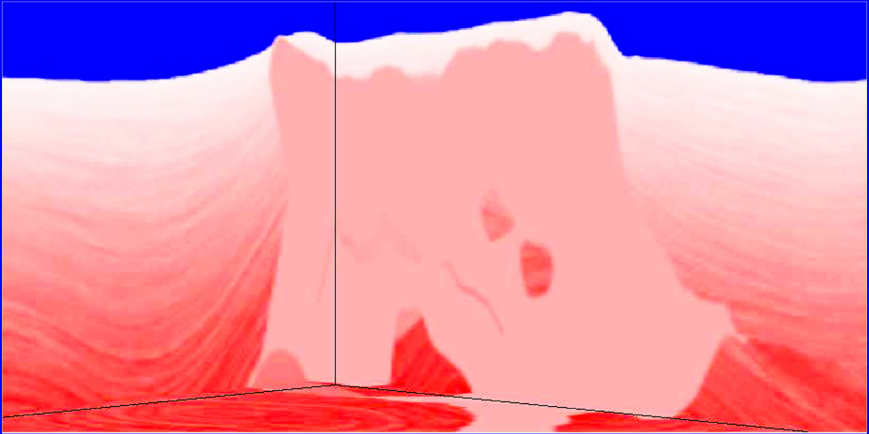
(a) Structure



(b) Morphed Stratigraphy

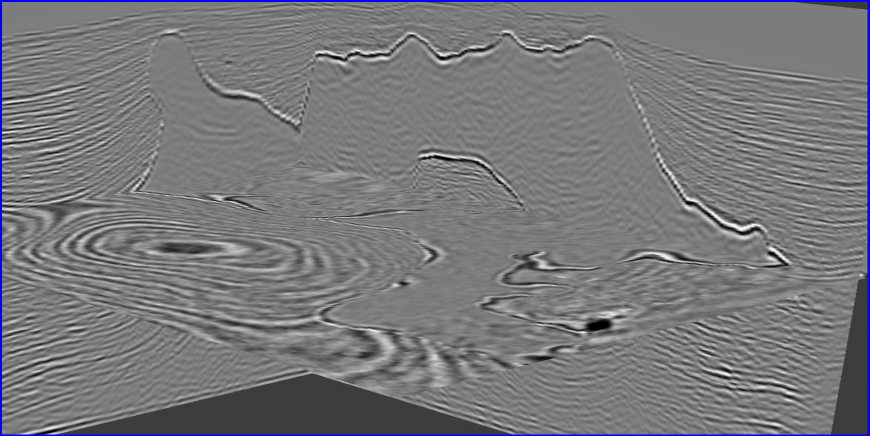
- Automatic estimation of horizons (a) (in red) on real data set.
- Inverse flattening of synthetic reflectivity in (b)
- Can be performed on ϵ, δ or γ
- Result is a stratigraphic/structural model

Combined Model (SEAM)



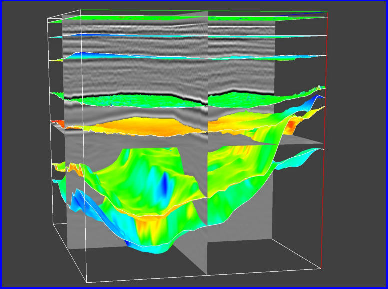
- SEAM (SEG) model constructed from logs and structural interpretation
- Realistic Gulf of Mexico salt model

Stratigraphic and Structural Image

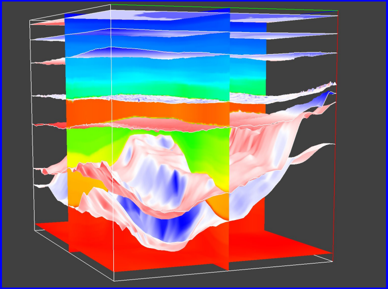


- Reverse time migration of the data synthesized over the SEAM model

Migration, surfaces, and V_p



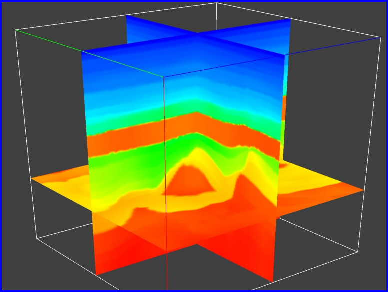
(a) Migration and Structural Surfaces



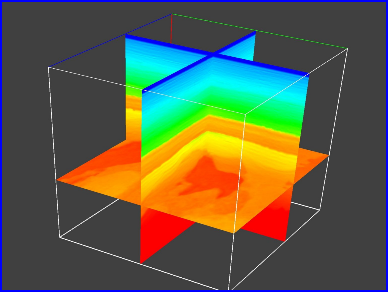
(b) V_p and Structural Surfaces

- Geologically consistent Earth model derived from interpreted structural surfaces and migration velocity analysis of 3D seismic measurements.

δ and ϵ components of complex Earth model



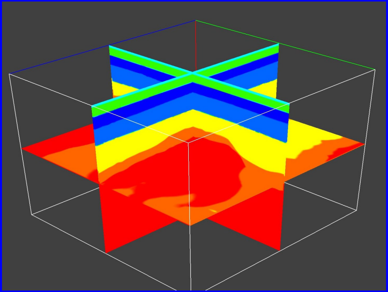
(a) Horizon based velocity



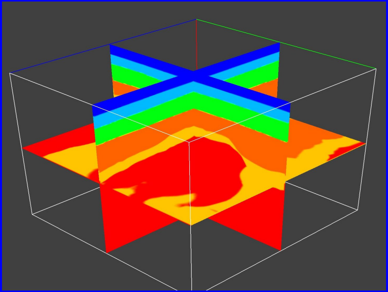
(b) ρv Reflectivity

- Horizon based background velocity in (a).
- Gardner-Gardner-Gregory ($\rho \approx \alpha V_p^{\beta}$) plus background image for reflectivity (b)

Reflectivity model



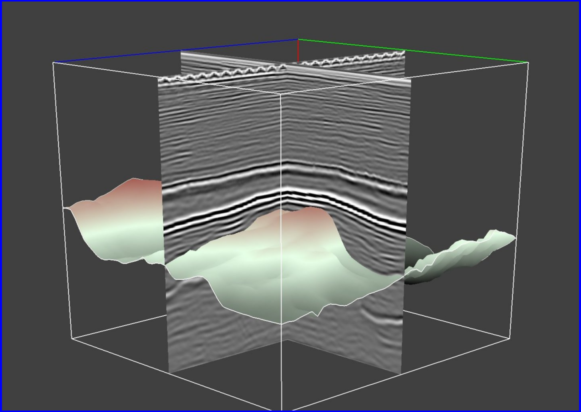
(a) δ



(b) ϵ

- δ and ϵ derived from interpreted seismic and borehole measurements.

Imaged VTI synthetic



- Imaged 3D wide azimuth synthetic data over Scott Earth model.
- Technology useful for advanced survey design, quantifying attributes · · ·

Questions?

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